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CORRECTION TO THE PAPER "FRACTIONAL INDICES OF LOG DEL PEZZO SURFACES"

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In the proof of Lemma 4.1 of the paper named above([1]) I committed an arithmetic error, because of which the entire proof of this lemma is invalid. The text must be corrected in the following way:

LEMMA 4.1. *Let* $\pi\colon \widetilde{X} \to X$ *be a minimal resolution of singularities of the log del Pezzo surface* X, *and let* $\rho(\widetilde{X}) = k$. *Then either*

(i) *there exists a birational morphism* $\widetilde{X} \to \mathbf{F}_n$ *and* $n > N$, *or*
(ii) *for any exceptional curve* C *on* $\widetilde{X}$ *the inequality* $0 > C^2 > A(k, N)$ *obtains, where* $A(k, N)$ *is a constant depending only on* k *and* N.

PROOF. By Proposition 1.3 the cone of effective curves on $\widetilde{X}$ is generated by finitely many curves. From the formula $K_{\widetilde{X}} = \pi^* K_X + \sum \alpha_i F_i$ it follows that the divisor $-MK_{\widetilde{X}}$ is effective for some positive integer M. Let C be an irreducible reduced curve with $C^2 < 0$. We now carry out the following procedure: if $l \neq C$ is an exceptional curve of genus 1 and $C \cdot l \leq 1$, then we contract this curve. We repeat this procedure several times until we obtain a morphism $f\colon \widetilde{X} \to S$ with the following properties: S is a nonsingular surface, $C_1 = f(C)$ is a nonsingular curve and for any curve $l \neq C_1$ of genus 1 we have $C_1 \cdot l \geq 2$.

If $S = \mathbf{P}^2$ or $\mathbf{F}_n$, $n \leq N$, then $C_1^2 \geq -N$ and $C^2 \geq C_1^2 - k \geq -N - k$. If $C_1^2 \geq -3$, then $C^2 \geq -3 - k$.

Now suppose $S \neq \mathbf{P}^2$, $\mathbf{F}_n$ and $C_1^2 \leq -4$. We prove that the divisor $2K_S + C_1$ is numerically effective. The cone of effective curves on the surface S is generated by finitely many curves; let $\{E_i\}$ be a minimal system of generators. If $K_S \cdot E_i \geq 0$ and $E_i \neq C_1$, then $(2K_S + C_1) \cdot E_i \geq 0$. If $K_S \cdot E_i < 0$ and $E_i \neq C_1$, then E_i is an exceptional curve of genus 1 and $(2K_S + C_1) \cdot E_i \geq 0$. Finally, $(2K_S + C_1) \cdot C_1 = 4p_\alpha(C_1) - 4 - C_1^2 \geq 0$.

Thus, the divisor $2K_S + C_1$ is numerically effective and the divisor $-MK_S =$

([1])Izv. Akad. Nauk SSSR Ser. Mat. **52** (1988), no. 6, 1288–1304=Math. USSR Izv. **33** (1989), no. 3, 613–629.

$f_*(-MK_{\widetilde{X}})$ is effective for some positive integer M. Therefore

$$-K_S \cdot (2K_S + C_1) \geq 0, \qquad -K_S \cdot (2K_S + C_1) = -2K_S^2 + 2 + C_1^2,$$
$$C^2 + k \geq C_1^2 \geq 2K_S^2 - 2 \geq 2K_{\widetilde{X}} - 2 = 2(10 - k) - 2$$

by Noether's formula. The lemma is proved.

Translated by B. SILVER