1. Let \( \{f_n\} \) be a sequence of differentiable functions on \((0, 1)\), converging uniformly to a differentiable function \(f\).

   (a) Give an example to show that the derivatives \(f'_n\) need not converge uniformly to \(f'\).

   (b) Show that if \(f'_n\) converges uniformly to some function \(g\), then \(f' = g\).

2. State and prove a Mean Value Theorem for a function \(f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}, n > 1\). Be sure to include all necessary hypotheses for the domain \(D\) and the function \(f\). (You may use without proof the mean value theorem for \(g : \mathbb{R} \rightarrow \mathbb{R}\)).

3. Prove that a continuous real-valued function on a compact metric space is uniformly continuous.

4. (a) Define the Lebesgue measure of a measurable set \(A \subseteq \mathbb{R}\).

   (b) Prove, using the definition, that the Lebesgue measure of \([0, 1]\) equals 1.

5. Let \(\phi(x)\) be a continuous function on \(\mathbb{R}\), with \(\int_{-\infty}^{\infty} \phi(x)dx = 1\) and \(\phi(x) = 0\) \(\forall |x| > 1\). Prove that for any continuous function \(f\) on \(\mathbb{R}\), \(\int_{-\infty}^{\infty} f(x)n\phi(nx)dx \rightarrow f(0)\) as \(n \rightarrow \infty\).

6. A probability measure \(\mu\) on \(\mathbb{R}\) is a positive Borel measure \(\mu\) with \(\mu(\mathbb{R}) = 1\). If \(\mu, \nu\) are probability measures on \(\mathbb{R}\), then so is \(\mu * \nu\), defined by \((\mu * \nu)(A) = \int\int \chi_A(s + t)d\mu(s)d\nu(t)\).

   (a) Verify that \(\mu * \nu\) is countably additive.

   (b) Verify carefully that \(\mu * \nu = \nu * \mu\).

7. Assume that all measures are positive and finite.

   (a) Prove that if \(\mu_1\) and \(\mu_2\) are each singular with respect to \(\nu\), then so is \(\mu_1 + \mu_2\).

   (b) Suppose that \(\mu\) is singular with respect to \(\nu\). Compute the Radon-Nikodym derivative of \(\mu\) with respect to \(\mu + \nu\).
(c) Suppose that $\mu$ is absolutely continuous with respect to $\nu$. In terms of the Radon-Nikodym derivative of $\mu$ with respect to $\nu$, compute the Radon-Nikodym derivative of $\mu$ with respect to $\mu + \nu$.

8. Let $X$ be a Banach space.

(a) Prove that the canonical mapping $X \rightarrow X^{**}$ is an isometry.

(b) Give an example of a Banach space $X$ which is not reflexive, and verify your example by computing $X^*, X^{**}$.

9. Prove that the Banach space $\ell^1$ is separable, and that the Banach space $\ell^\infty$ is not separable.