By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): Key Name (print): ____________________

Student Number: ____________________

Instructor's Name: ____________________ Class Time: ________________

Problem Number	Points Possible	Points Made
1	18	
2	26	
3	16	
4	20	
5	22	
6	20	
7	15	
8	20	
9	15	
10	15	
11	15	
12	15	
13	30	
14	10	
15	15	
Total:	272	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a **TI-30XS Multiview** calculator. No other calculators are permitted, and sharing of calculators is not permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

1. Determine the following limits. If you answer with ∞ or $-\infty$, briefly explain your thinking. Print your final answer in the box provided.

______ (a) [5 pts] $\lim_{x\to 2}(3x^2 + 7x - 5)$

$= 3\cdot 2^2 + 7\cdot 2 - 5$
$= 12 + 14 - 5$
$= 26 - 5$

answer: 21

______ (b) [5 pts] $\lim_{x\to 1^-} \frac{2x}{x-1}$

$2x \to 2$ as $x \to 1^-$
$x - 1 \to 0$ and is negative as $x \to 1^-$

answer: $-\infty$

______ (c) [8 pts] $\lim_{x\to\infty} \frac{\ln(5x)}{x^3+1}$ IF $\frac{\infty}{\infty}$

$\stackrel{LH}{=} \lim_{x\to\infty} \frac{\frac{1}{5x}\cdot 5}{3x^2}$

$= \lim_{x\to\infty} \frac{1}{x}\cdot\frac{1}{3x^2}$

$= \lim_{x\to\infty} \frac{1}{3x^3}$

$= 0$

answer: 0

2. Determine the first derivative of each of the following functions. Print your answer in the box provided. *You do not have to simplify your answers or explain your steps.*

______ (a) [4 pts] $f(x) = 8x^3 - 15x + 12$

$f'(x) = 24x^2 - 15$

______ (b) [6 pts] $g(t) = \dfrac{\sin(t)}{t}$

$g'(t) = \dfrac{t\cos(t) - \sin(t)}{t^2}$

______ (c) [6 pts] $f(x) = \dfrac{e^x}{2x+1}$

$f'(x) = \dfrac{(2x+1)(e^x) - e^x \cdot 2}{(2x+1)^2} = \dfrac{e^x(2x-1)}{(2x+1)^2}$

______ (d) [10 pts] $h(x) = (4x-3)^2 \arctan(x)$

$h'(x) = (4x-3)^2 \cdot \dfrac{1}{1+x^2} + \arctan(x) \cdot 2(4x-3) \cdot 4$

$h'(x) = \dfrac{(4x-3)^2}{1+x^2} + 8(4x-3)\arctan(x)$

3. (a) [8 pts] Determine $\frac{dy}{dx}$ for the equation $y^3 - x^4 y = 6$. Print your answer in the box provided. You do not have to simplify your answer.

$$3y^2 \frac{dy}{dx} - \left(x^4 \frac{dy}{dx} + y \cdot 4x^3\right) = 0$$

$$3y^2 \frac{dy}{dx} - x^4 \frac{dy}{dx} - 4x^3 y = 0$$

$$3y^2 \frac{dy}{dx} - x^4 \frac{dy}{dx} = 4x^3 y$$

$$\frac{dy}{dx}\left(3y^2 - x^4\right) = 4x^3 y$$

$$\frac{dy}{dx} = \frac{4x^3 y}{3y^2 - x^4}$$

(b) [8 pts] Determine an equation of the tangent line to the curve $y^3 - x^4 y = 6$ at the point $(1, 2)$.

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{4 \cdot 1 \cdot 2}{3 \cdot 4 - 1} = \frac{8}{11}$$

Equation: $y - 2 = \frac{8}{11}(x - 1)$

4. Determine the following indefinite integrals. Print your answer to each part in the box provided.

(a) [4 pts] $\int \left(-4x^7 + 8x^5 + 12\right) dx$

$$= -\frac{4}{8}x^8 + \frac{8}{6}x^6 + 12x + C$$

Final answer: $-\frac{1}{2}x^8 + \frac{4}{3}x^6 + 12x + C$

(b) [6 pts] $\int \left(\sec^2(t) + \frac{1}{t}\right) dt$

Final answer: $\tan(t) + \ln|t| + C$

(c) [10 pts] $\int \frac{x^4}{\sqrt{x^5+3}} dx$

$$= \int (x^5+3)^{-1/2} \cdot x^4\, dx$$

$$= \frac{1}{5}\int (x^5+3)^{-1/2} \cdot 5x^4\, dx$$

$u = x^5+3$
$du = 5x^4\, dx$

$$= \frac{1}{5}\int u^{-1/2}\, du$$

$$= \frac{1}{5} \cdot 2u^{1/2} + C$$

Final answer: $\frac{2}{5}\sqrt{x^5+3} + C$

5. Evaluate the following definite integrals. Print your answer in the box provided.

(a) [6 pts] $\displaystyle\int_1^8 \left(x^{2/3} - \frac{1}{x^{4/3}}\right)\,dx$

$= \int_1^8 (x^{2/3} - x^{-4/3})\,dx$

$= \left[\frac{3}{5}x^{5/3} + 3x^{-1/3}\right]_1^8$

$= \frac{3}{5}(8)^{5/3} + 3\cdot 8^{-1/3} - \left(\frac{3}{5}+3\right)$

$= \frac{3\cdot 32}{5} + \frac{3}{2} - \frac{3}{5} - \frac{6}{2}$

$= \frac{96}{5} - \frac{3}{5} - \frac{3}{2}$

$= \frac{93}{5} - \frac{3}{2}$

Value: $= \frac{186}{10} - \frac{15}{10} = \frac{171}{10} = 17.1$

(b) [6 pts] $\displaystyle\int_0^{1/2} \frac{-1}{\sqrt{1-x^2}}\,dx$

$= [\arccos(x)]_0^{1/2}$

$= \arccos(1/2) - \arccos(0)$

$= \frac{\pi}{3} - \frac{\pi}{2}$

$= [-\arcsin(x)]_0^{1/2}$

$= -\arcsin(1/2) + \arcsin(0)$

$= -\frac{\pi}{6} + 0$

Value: $-\frac{\pi}{6}$

(c) [10 pts] $\displaystyle\int_0^{\pi/4} \sin(4x)e^{\cos(4x)}\,dx = \int_1^{-1} -\frac{1}{4}e^u\,du = \left[-\frac{1}{4}e^u\right]_1^{-1}$

$u = \cos(4x)$

$du = -4\sin(4x)\,dx$

$u(0) = 1$

$u(\pi/4) = -1$

Value: $-\frac{1}{4}e^{-1} - \left(-\frac{1}{4}e^{1}\right) = \frac{-1}{4e} + \frac{e}{4}$

6. (a) [5 pts] State the limit definition of the derivative of $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

(b) [10 pts] Use the limit definition of the derivative to show that the derivative of $f(x) = 12x - 2x^2$ is $f'(x) = 12 - 4x$. (You will receive 0 points for using the power rule.)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{12(x+h) - 2(x+h)^2 - 12x + 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{12x + 12h - 2(x^2 + 2xh + h^2) - 12x + 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{12x + 12h - 2x^2 - 4xh - 2h^2 - 12x + 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{12h - 4xh - 2h^2}{h}$$

$$= \lim_{h \to 0} (12 - 4x - 2h)$$

$$= 12 - 4x$$

(c) [5 pts] Determine all values of x for which the graph of $f(x) = 12x - 2x^2$ has a horizontal tangent line.

$$12 - 4x = 0$$

$$12 = 4x$$

$$\boxed{3 = x}$$

7. [15 pts] Determine the absolute maximum and absolute minimum values of $f(x) = 2x\sqrt{9-x}$ on the interval $[-1, 9]$.

$$f'(x) = 2x \cdot \frac{1}{2}(9-x)^{-1/2} \cdot -1 + \sqrt{9-x} \cdot 2$$

$$= \frac{-x}{\sqrt{9-x}} + \frac{2\sqrt{9-x}}{1} \cdot \frac{\sqrt{9-x}}{\sqrt{9-x}}$$

$$= \frac{-x + 2(9-x)}{\sqrt{9-x}}$$

$$= \frac{-x + 18 - 2x}{\sqrt{9-x}}$$

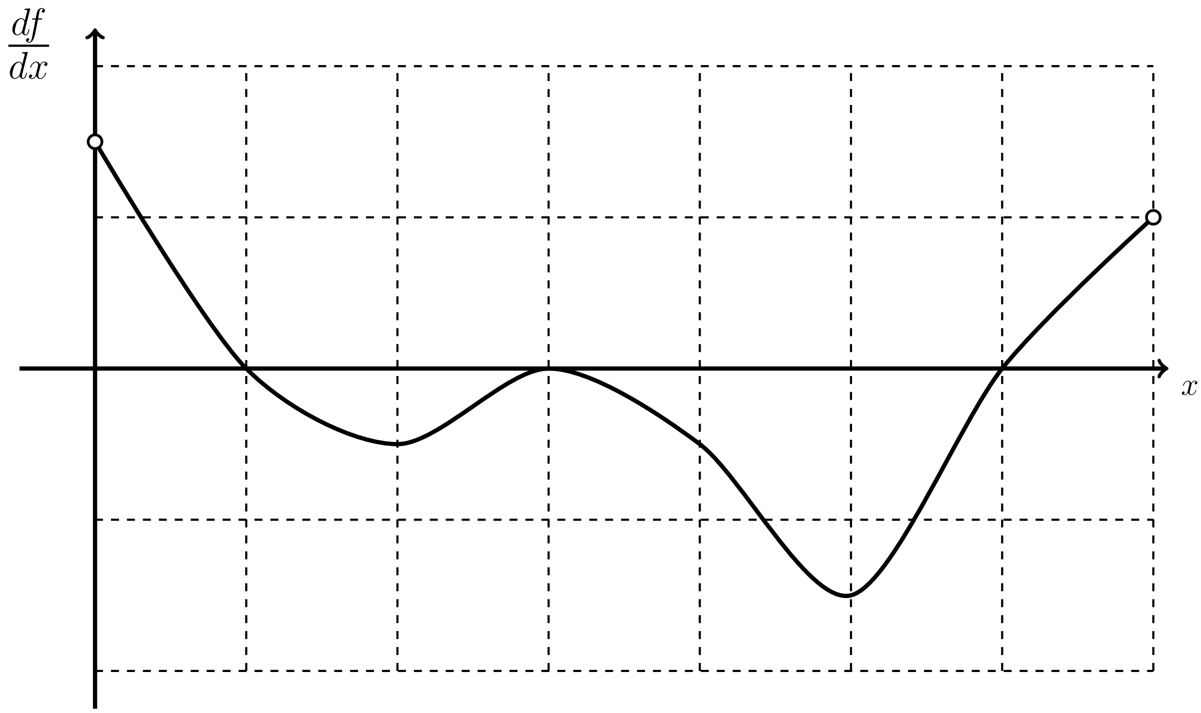
$$= \frac{18-3x}{\sqrt{9-x}}$$

f' dne: $\cancel{x=9}$ not a critical number (domain)

$f'(x)=0$: $x = 6$

x	$f(x)$
-1	$-2\sqrt{10}$ ← absolute min is $-2\sqrt{10}$
6	$12\sqrt{3}$ ← absolute max is $12\sqrt{3}$
9	0

8. The graph below is the graph of **the derivative of** $f(x)$. Use it to answer the questions that follow. The grid lines are one unit apart, and the domain of f is $(0, 7)$.



(a) [5 pts] Determine all critical numbers (critical points) of f.

$x = 1, 3, 6$

(b) [5 pts] Determine the intervals on which f is increasing.

$(0,1] \cup [6,7)$

also accepted: $(0,1) \cup (6,7)$

(c) [5 pts] Determine all values of x at which f has a local minimum.

f' changes sign from $-$ to $+$ at $x = 6$

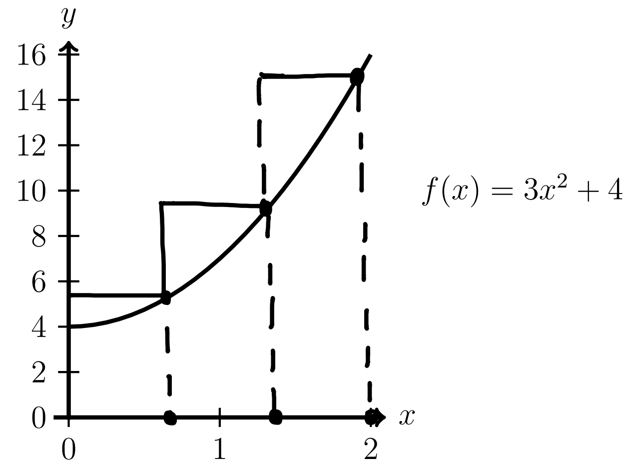
(d) [5 pts] Determine the intervals on which f is concave up.

f is concave up where f' is increasing:

$[2,3] \cup [5,7]$

also accepted: $(2,3) \cup (5,7)$

9. For this problem, use $f(x) = 3x^2 + 4$ on the interval $[0, 2]$. Its graph is provided to the right.



(a) [5 pts] Determine a Riemann sum for f on the interval $[0, 2]$ using 3 subintervals of equal width and using right endpoints on each subinterval.

$\Delta x = \frac{2-0}{3} = \frac{2}{3}$ $[0, \frac{2}{3}][\frac{2}{3}, \frac{4}{3}][\frac{4}{3}, 2]$

Rie. sum: $f(2/3) \cdot 2/3 + f(4/3) \cdot 2/3 + f(2) \cdot 2/3$

$= (3(\frac{2}{3})^2+4) \cdot 2/3 + (3(4/3)^2+4) \cdot \frac{2}{3} + (3(2)^2+4) \cdot 2/3$

(b) [5 pts] Is your Riemann sum above an over- or under-estimate of the integral $\int_0^2 f(x)\,dx$? Explain how you can tell, *without doing any calculations or working out the answers*, whether it's an over-estimate or an under-estimate. (You may want to illustrate the Riemann sum on the graph of f provided above.)

It's an over-estimate because f is increasing on $[0,2]$

or: ... because the rectangles above contain more area than the region under the curve.

(c) [5 pts] Use summation (sigma) notation to write an expression for a Riemann sum for f on the interval $[0, 2]$ using n subintervals of equal width and using right endpoints on each subinterval. You do not have to work out the value of the sum, but your sum should involve only $\sum_{k=1}^{n}$, the variables k and n, and numbers.

$\Delta x = \frac{2}{n}$

$c_k = \frac{2k}{n}$

$f(c_k) = 3(c_k)^2 + 4 = 3\left(\frac{2k}{n}\right)^2 + 4$

Rie. sum $= \sum_{k=1}^{n} \left[3\left(\frac{2k}{n}\right)^2 + 4\right] \cdot \frac{2}{n} = \sum_{k=1}^{n} \left(\frac{12k^2}{n^2} + 4\right) \cdot \frac{2}{n}$

$= \boxed{\sum_{k=1}^{n} \left(\frac{24k^2}{n^3} + \frac{8}{n}\right)}$

10. Use the values of the given definite integrals to determine the quantities below.

$$\int_1^7 f(x)\ dx = -8, \qquad \int_3^7 f(x)\ dx = 12, \qquad \int_1^7 g(x)\ dx = 9$$

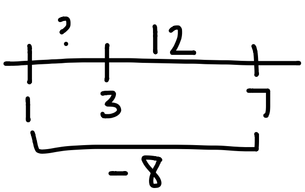
_____ (a) [5 pts] $\int_1^7 (2f(x) - 5g(x))\ dx$

$= 2(-8) - 5(9)$

$= -16 - 45$

$= -61$

_____ (b) [5 pts] $\int_1^3 f(x)\ dx = -20$



$? + 12 = -8$

$? = -20$

_____ (c) [5 pts] $\int_1^7 (g(t) - t^2)\ dt$

$= \int_1^7 g(t)\,dt - \int_1^7 t^2\,dt$

$= 9 - \left[\frac{1}{3}t^3\right]_1^7$

$= 9 - \left(\frac{1}{3}\cdot 7^3 - \frac{1}{3}\cdot 1^3\right) \longleftarrow$ ok final answer

$= \frac{27}{3} - \frac{343}{3} + \frac{1}{3} = \frac{-315}{3} = 105$

11. The charts below contain information about a function f and its derivative. Assume that f is differentiable on $[-2, 1]$. Use the charts to answer the questions that follow.

x	-2	-1	0	1
$f(x)$	3	2	0	-1

x	-2	-1	0	1
$f'(x)$	$-\frac{1}{8}$	$-\frac{1}{3}$	-1	0

______ (a) [5 pts] Determine the linearization of f at $x = -1$.

$$L(x) = f(-1) + f'(-1)(x+1)$$
$$= \boxed{2 + -\tfrac{1}{3}(x+1)} \leftarrow \text{ok final answer}$$
$$= 2 - \tfrac{1}{3}x - \tfrac{1}{3}$$
$$= \tfrac{5}{3} - \tfrac{1}{3}x$$

(b) [5 pts] Use your linearization above to estimate the value of $f(-1.5)$.

______

$$f(-1.5) \approx L(-1.5) = \boxed{2 - \tfrac{1}{3}(-1.5+1)} \leftarrow \text{ok final answer}$$
$$= 2 - \tfrac{1}{3}\left(-\tfrac{1}{2}\right)$$
$$= 2 + \tfrac{1}{6}$$
$$= \tfrac{13}{6}$$
$$= 2.1\overline{6}$$

(c) [5 pts] Suppose you also know that f' is continuous on $[-2, 1]$. Explain why the graph of f must have an inflection point somewhere in the interval $[-2, 1]$.

______

Based on the chart for f', there must be an inflection point in $[-2,1]$ since f' changes from decreasing to increasing (at least once).

12. [15 pts] A diesel truck develops an oil leak. The oil drips onto the dry ground in the shape of a circular puddle. Assuming that the leak begins at time $t = 0$ and that the radius of the oil slick increases at a constant rate of .05 meters per minute, determine the rate of change of the area of the puddle 4 minutes after the leak begins.

goal: $\frac{dA}{dt}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dr}{dt} = .05$ meters/minute

$r = \frac{.05 \text{ meters}}{1 \text{ min}} \times 4 \text{ minutes} = 0.2\text{m}$ since $\frac{dr}{dt}$ is <u>constant</u>

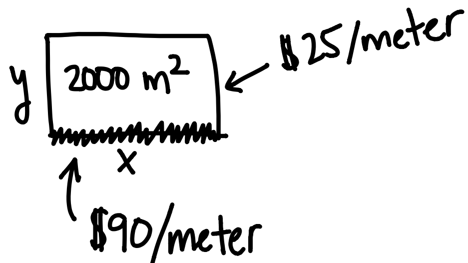
$\frac{dA}{dt} = 2\pi(0.2)(.05)$ square meters/minute

$= .02\pi$ square meters/minute

$\approx .06283$ square meters/minute

13. A landscape designer plans to construct a rectangular garden whose area is 2000 square meters. One side will consist of a wrought iron fence which costs \$90 per meter. The remaining three sides will be constructed from chain link fence costing \$25 per meter.

_______ (a) [15 pts] Determine a function for the total cost $C(x)$ of the garden, where x is the length of wrought iron fence used (in meters).



$$C(x) = 90x + 25\left(\frac{2000}{x}\right) + 25x + 25\left(\frac{2000}{x}\right)$$

$$C(x) = 115x + \frac{100\,000}{x}$$

$$x \cdot y = 2000 \qquad y = \frac{2000}{x}$$

_______ (b) [15 pts] What dimensions of the garden will minimize the total cost? Use calculus techniques to show that the dimensions result in the minimum possible cost.

domain: $(0, \infty)$

$$C'(x) = 115 - \frac{100\,000}{x^2}$$

C' dne: none $\quad C' = 0$: $\quad 115 = \dfrac{100\,000}{x^2}$

$$x^2 = \frac{100\,000}{115}$$

$$x^2 = \frac{20000}{23}$$

$$x = \sqrt{\frac{20\,000}{23}} = 100 \cdot \sqrt{\frac{2}{23}}$$

Note: The first derivative test is also fine to use. see the last page.

Second derivative test: $\quad C''(x) = \dfrac{200\,000}{x^3}$

$$C''\left(\sqrt{\frac{20000}{23}}\right) = \frac{200\,000}{\sqrt{\frac{20000}{23}}^{\,3}} = 2000 \cdot \sqrt{\frac{23}{2}} > 0$$ so $C(x)$ has a local min at $x = 100 \cdot \sqrt{2/23}$

Since there is only one critical number in $(0, \infty)$, the local min is an absolute min. The dimensions are $x = 100 \cdot \sqrt{\frac{2}{23}} \approx 29.4884$ meters

and $y = \dfrac{2000}{100\sqrt{2/23}} \approx 67.8233$ meters.

14. [10 pts] Let $y = \ln(x)$. Show that $\dfrac{dy}{dx} = \dfrac{1}{x}$ by solving the equation $y = \ln(x)$ for x and then using implicit differentiation. Your final answer should be $\dfrac{dy}{dx}$, given as a function of x.

$y = \ln(x)$

$e^y = x$ (solve for x)

$e^y \dfrac{dy}{dx} = 1$ (differentiate both sides with respect to x)

$\dfrac{dy}{dx} = \dfrac{1}{e^y}$ (solve for $\dfrac{dy}{dx}$)

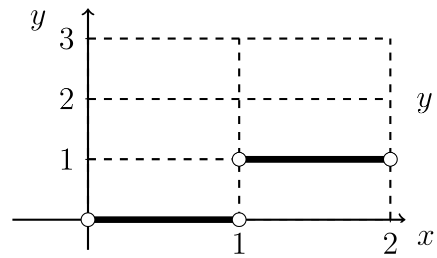
$\dfrac{dy}{dx} = \dfrac{1}{x}$ (sub in x for e^y since $e^y = x$)

15. Information about a function, f, and its derivative is given below. Use the information to answer the questions that follow.

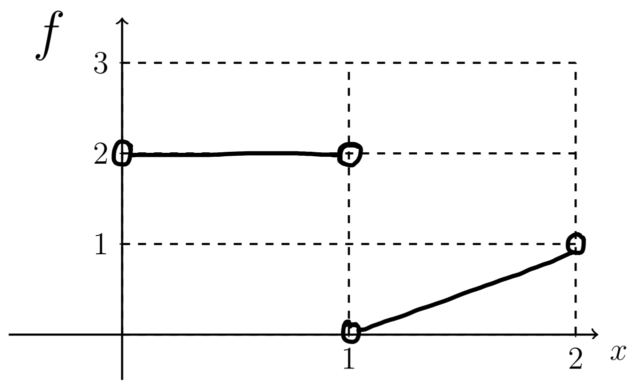
Information about f:

$$\lim_{x\to 0^+} f(x) = 2,$$
$$\lim_{x\to 2^-} f(x) = 1,$$

This is the graph of the **derivative** of f:



(a) [5 pts] Make a rough sketch of the graph of $y = f(x)$. (Hint: Think about slopes.)



function values at $x = 0, 1, 2$ are not needed

* Does your graph pass the vertical line test at $x=1$?

(b) [5 pts] Determine $\lim_{x\to 1^-} f(x)$.

2

(c) [5 pts] Determine $\lim_{x\to 1^+} f(x)$.

0

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): __________ Instructor (print): __________ Time: __________

#13 – 1^{st} derivative test

$$C'(x) = 115 - \frac{100{,}000}{x^2} = \frac{115x^2 - 100{,}000}{x^2}$$

critical number: $x = 100\sqrt{2/23} \approx 29.49$

	$(0, 100\sqrt{2/23})$	$(100\sqrt{2/23}, \infty)$
sign of C'	$C'(1) = 115 - 100000$ $\ominus$	$C'(100) = \frac{115(100)^2 - 100000}{100^2} = \frac{115 \cdot 10000 - 100000}{100^2}$ $\oplus$
behavior of C	$\searrow$	$\nearrow$

Therefore $C(x)$ has a local min at $x = 100\sqrt{2/23}$

Since there is only one critical number in $(0, \infty)$, the local min is an absolute min. The dimensions are $x = 100 \cdot \sqrt{\frac{2}{23}} \approx 29.4884$ meters

and $y = \frac{2000}{100\sqrt{2/23}} \approx 67.8233$ meters.