Department Syllabus for MATH 3200

Course Description (to be listed in UGA Bulletin)
This course covers mathematical reasoning and writing mathematical proofs, the two essential skills for success in upper division course work in mathematics. Topics include logic, integers and induction, sets and relations, equivalence relations, and functions (including injectivity and surjectivity).

Student Learning Outcomes
At the end of the semester, a successful student will be able to:

1. Define and correctly use basic vocabulary associated with the following topics:
    a. Logic
    b. The real numbers, especially the integers
    c. Induction
    d. Set Theory
    e. Relations, especially equivalence relations
    f. Functions
2. Generate examples and non-examples of mathematical objects associated with the topics above.
3. Use correct mathematical notation associated with the topics above.
4. Formulate logically sound arguments using style conventions common in mathematical practice.
5. Identify an appropriate proof technique for an assigned proof.
6. Write mathematically valid proofs using the following techniques:
    a. Direct proof
    b. Biconditional proof
    c. Proof by cases
    d. Proof by contrapositive
    e. Proof by contradiction
    f. Induction
7. Write mathematically valid proofs in the following subject areas:
    a. The real numbers, especially the integers
    b. Sets
    c. Relations, especially equivalence relations
    d. Functions

A few notes on breadth and depth
<u>Logic</u> At the end of the logic unit, the successful student will be able to:

- Manipulate implications (negate, rephrase, find contrapositive and converse, etc.)
- Manipulate quantified statements (negate, rephrase, etc.)

- Interpret a biconditional by decomposing it into two conditional statements

Sets At the end of the set theory unit, the successful student will be able to:
- Determine whether one set is a subset of another, and prove their assertion
- Prove two sets are equal by showing each is a subset of the other
- Prove two sets are equal by showing step-by-step manipulations of the set definition using setbuilder notation
- When sets are given in terms of operations on other sets: prove two sets are equal by performing operations (DeMorgan's laws, distributive property, etc.) on one set until you obtain the other one

Functions At the end of the functions unit, the successful student will be able to:
- Determine whether or not a given function is (or is not) injective, surjective, bijective, and prove their assertion
- Compute simple preimages or images of sets, especially for continuous functions of real numbers
- Construct simple proofs involving preimages and images of sets under functions
- Construct proofs involving compositions of functions

Relations At the end of the relations unit, the successful student will be able to:
- Determine whether or not a relation is (or is not) an equivalence relation, and prove their assertion
- Determine equivalence classes for a given equivalence relation, especially for relations with modular congruence.

Integers At the end of the course, the successful student will be able to:
- Prove statements about integer parity
- Write induction proofs
- Optional: write strong induction proofs
- Optional: simplify calculations in modular arithmetic
- Optional: Euclidean algorithm for determining GCD

Number systems At the end of the course, the successful student will be able to:
- Optional: perform arithmetic calculations using complex numbers
- Optional: write proofs involving basic properties of rational numbers

Commonly used textbooks

Note: You may need more than one textbook to cover all of the syllabus topics.
- Ron Taylor, *Introduction to Proof*, Available at jiblm.org as a free PDF
- Chartrand, Polimeni, and Zhang, *Mathematical Proofs: A Transition to Advanced Mathematics* (3rd Edition)
- Houston, *How to Think Like a Mathematician: A Companion to Undergraduate Mathematics*

The table below includes chapter references for the recommended topics.

Topic	Taylor	Chartrand	Houston
Logic	1	2, 7	6-11
Sets	4	1	1, 20
Functions	5	9	1, 30
Relations	5	8	31
Integers	2	3, 4, 11	20, 22-24, 26, 27-29
Number Systems			

The following table includes chapter references for the methods of proof:

Proof Method	Taylor	Chartrand	Houston
Direct Proof	2	3	20
Biconditional Proof	2		20
Proof by Cases	2	3	22
Proof by Contrapositive	2	3	26
Proof by Contradiction	2	5	23
Induction	3	6	24

Classroom Style
This material is well suited to an active learning format. Students can generally read the textbook in advance so that you can use class time for activities. Sample class activities include group work, problem presentations, writing-to-learn activities, and peer review.

Assessment
Students need frequent feedback on their writing; group work provides an opportunity for students to receive feedback from their peers. Collect homework frequently and make sure that students receive clear feedback on how to improve their writing. It's a good idea to quiz students on definitions to make sure that they have a foundation for their mathematical thinking.

Thinking Ahead
Near advising and registration, encourage your students to take more math courses, and recommend courses for their consideration.