

Final Exam Review

Fall 2018

MATH 2250

University of Georgia

EXAM DATE/TIME: Thursday, December 6, 7:00 p.m. - 10:00 p.m.

Location announced by your professor

Definitions and Theorems to State:

- The (limit) definition of the derivative of $f(x)$
- The definition of continuity at $x = a$

Theorems to Use but not State (if you use one, say you are using it):

- The Extreme Value Theorem (Closed Interval Method)
- First derivative test for local extrema
- L'Hopital's rule
- The relationship of continuity to differentiability
- Second derivative test for local extrema
- Fundamental Theorems of Calculus (Part 1 and Part 2)
- Intermediate Value Theorem

Properties you will be responsible for:

- Properties of logarithmic and exponential functions
- other precalculus-level formulas (in addition to those listed below)

Limits:

- Be able to find the limits and one-sided limits of functions (even if not continuous), both analytically and graphically
- Find limits that approach infinity or have an infinite limit
- Determine horizontal asymptotes and vertical asymptotes of a function; justify your answer using one or more limits
- Be able to use L'Hopital's Rule to find limits (and identify and state the appropriate indeterminate forms that allow you to do so)
- Applications of continuity, including the Intermediate Value Theorem
- Verify continuity (analytically and graphically)
- Determine intervals on which a function is continuous
- Be able to "repair" a removable discontinuity by (re)defining the function at that x -value
- Determine the value of a parameter that makes a piecewise function continuous where the two pieces meet

Derivatives:

- Be able to find the derivative $f'(x)$ from the definition
- Be able to use rules to find the derivative; know all rules from back of book through inverse trig function (no hyperbolic or parametric, no $\operatorname{arcsec}(x)$, $\operatorname{arccot}(x)$, or $\operatorname{arccsc}(x)$)
- Implicit differentiation
- Be able to compute derivatives at specific points using limited information (e.g. a table)
- Be able to find an equation of the tangent line at a point

- Be able to understand/interpret the slope of a function
- Logarithmic differentiation

Proof-based Problems:

- Use differentiation of the appropriate inverse function to verify the differentiation rule for $\ln(x)$
- Use differentiation of the appropriate inverse function to verify the differentiation rule for $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$ (including an appropriate right triangle diagram or a Pythagorean identity)

Applications of Derivatives

- Applications involving a tangent line
- Be able to find and use the linearization
- Be able to find and use the differential
- Position, displacement, velocity, acceleration problems
- Applied rates of change other than position, velocity, and acceleration
- Related rates
- Understand the relationship between (first and second) derivatives and curve behavior; curve sketching from derivative information
- Determine all extrema of a function on a closed interval
- Applied optimization (open and/or closed interval); justify that you have a max or min

Integration

- Antiderivatives: most general antiderivative as well as initial value problems
- Understand the definite integral as (signed) area
- Apply properties of the definite integral
- Be able to use the definite integral to compute and interpret
 - signed area
 - total area
 - area between two curves
 - average value of a function
- Estimate a definite integral using well-chosen sums with a small number of rectangles (left, right, midpoint), and interpret your answer
- Express a right endpoint Riemann sum with N rectangles of equal width in summation form, using only the summation symbol, k , N , and numbers
- Compute a definite integral:
 - by interpreting it as area
 - by Evaluation Theorem (FTC 2)
 - by integration via substitution

Terminology to be familiar with (in addition to terminology listed in sections above):

- average rate of change/secant slope, average velocity
- instantaneous rate of change/tangent slope

- average value of a function
- tangent lines and linearization of a function at a point
- domain
- critical points (critical numbers), inflection points
- increasing, decreasing, concave up, concave down
- local (relative) extrema
- absolute (relative) extrema

Penalties (approximately 20% of problem's points value for each issue):

- Improper use of $+C$ or missing $+C$
- Improper use of limit notation
- Improper use of integral or sigma
- Improper use of "=" (like $y = \cos^2(x) = -2 \cos(x) \sin(x)$)
- Improper algebraic notation (missing parentheses, incorrect variable name, etc.)

Remarks for students:

- Problems may combine multiple topics/techniques.
- You do not have to simplify your answers.
- Calculator **TI 30XS Multiview only!** No other calculators are allowed, and sharing of calculators is not allowed.
- Final answers are preferred in symbolic form (like $\sqrt{3}$ or e^2) but a FINAL decimal approximation must be correct to 3 decimal places.
- You will leave your backpacks at the front of the room; a backpack that rings or buzzes will be taken out to the hallway and left there.
- No smart watches are allowed during the exam; your cell phone may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.
- No bathroom breaks unless you have a documented medical condition. Let your instructor know in advance.

Formulas to Remember

- Distance between (x_1, y_1) and (x_2, y_2) : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Triangles and Trig
 - perimeter (add up side lengths)
 - area: $A = \frac{1}{2}bh$
 - Be able to use properties of similar triangles.
 - Pythagorean Theorem for right triangles: $a^2 + b^2 = c^2$
 - right triangles and acute angle trig: SOH-CAH-TOA

$\tan(x) = \frac{\sin(x)}{\cos(x)}$	$\cot(x) = \frac{\cos(x)}{\sin(x)}$
$\csc(x) = \frac{1}{\sin(x)}$	$\sec(x) = \frac{1}{\cos(x)}$

- $\sin^2(x) + \cos^2(x) = 1$

– Your trig differentiation formulas assume that your angle is in radians. (Why?)

- Circles

- area: $A = \pi r^2$

- circumference: $C = 2\pi r$

- Equation of the circle of radius r centered at (h, k) : $(x - h)^2 + (y - k)^2 = r^2$

- Rectangles

- area: $A = lw$

- perimeter: $P = 2l + 2w$

- Cylinder

- volume: $V = \pi r^2 h$

- surface area: $S = 2\pi r^2 + 2\pi r h$ (includes base and lid)

- Rectangular prisms

- volume: $V = lwh$

- surface area: $S = 2lw + 2wh + 2lh$ (includes top and base)