Sponsored by: UGA Math Department and UGA Math Club

TEAM ROUND / 1 HOUR / 210 POINTS
October 24, 2015

Problem 1 (Mathematics for fun and profit)**.** Here's how the Big Bucks Lottery works. When you buy a lottery ticket for \$1, you get to choose 3 different numbers from $\{1, 2, 3, 4, 5, 6, 7\}$. Once you've bought all the tickets you want, the lottery company randomly chooses 3 distinct numbers from $\{1, 2, 3, 4, 5, 6, 7\}$. For each of your tickets that matches all three numbers (jackpot!) you win \$10. For each of your tickets that matches exactly 2 numbers, you win \$3.

(a) (35 points) If you buy exactly one of every possible ticket, what will your profit be?

(b) (35 points) What is the smallest number of tickets you can buy and still be guaranteed to make a (positive!) profit?

Problem 2 (A binary word problem)**.** The *Thue–Morse sequence* $t_0, t_1, t_2, \ldots$ is a sequence of 0s and 1s defined by the rule

$$t_n = \begin{cases} 0 & \text{if } n \text{ has an even number of 1s in its binary expansion,} \\ 1 & \text{otherwise.} \end{cases}$$

For example, $t_0 = 0$ and $t_{13} = 1$. If the terms of the sequence are concatenated, one obtains an infinite "word" in the letters 0 and 1 which begins

$$0110100110010110\ldots,$$

where we take the starting "letter" to be $t_0 = 0$. How many occurrences of the string 11 are there in the initial segment

$$t_0 t_1 \dots t_{2014} t_{2015}?$$

In other words, for how many integers $0 \le n < 2015$ is $t_n = t_{n+1} = 1$?

Problem 3 (Be careful or you'll lose a digit!)**.** Dan D. Man (the D stands for "Digit") tabulates the leading decimal digits of each of the 2015 numbers $3^0, 3^1, \dots, 3^{2014}$. He observes that 3^{2014} has leading digit 8 and that the digit 9 appears 93 times as the leading digit. If A is the number of times that 1 appears the leading digit, and B the number of times that 2 appears, find $A + B$.

Authors. Problems and solutions were written by Mo Hendon and Paul Pollack.

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Team ID:

Team name:

Answer 1:

Answer 2:

Answer 3: