Sponsored by: UGA Math Department and UGA Math Club

Team Round / 45 min / 210 points
October 13, 2007

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1. (Platonic solids and plane tilings) How many triples of positive integers (p, q, r) are there such that

(a) Each of p, q, and r is at least 2,

(b) At most one of p, q, and r equals 2, and

(c)
$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} \geq 1?$$

Note that in this problem, triples are *unordered*; for example, (p, q, r) and (q, p, r) count as different triples if $p \neq q$.

Problem 2. (M-triples) How many triples of *positive integers* (a, b, c) are there with $a \leq b \leq c \leq 100$ that satisfy the equation $a^2 + b^2 + c^2 = 3abc$?

Note that in this problem, triples are *ordered*; we insist that $a \leq b \leq c$.

Problem 3. (Holy polyhedron) There exists a (non-convex) polyhedron with exactly one hole, such that every pair of faces shares exactly one edge. How many vertices does this polyhedron have?

Authors. Written by Boris and Valery Alexeev.

RETURN THIS SHEET

Team ID:

Team name:

Answer 1:

Answer 2:

Answer 3: