KOSSACK EXAM
March 30, 2006

Directions: You have two hours to do this exam. The use of calculators is not allowed. Show your work and reasoning. Do as many problems as you can.

1. Evaluate the following limits:
   a. $\lim_{x\to 1} \dfrac{\tan(x^2-1)}{x^2+x-2}$
   b. $\lim_{x\to\infty} \left(\sqrt{x^2+5x+2}-x\right)$
   c. $\lim_{n\to\infty} \left(\dfrac{n}{n-1}\right)^n$

2. Find all values of the constant c so that the curves $y = cx^2$ and $y = \ln x$ have exactly one point in common.

3. Evaluate, simplifying your answers as much as possible.
   a. $\displaystyle\int_0^1 \frac{\arctan x}{1+x^2}\,dx$
   b. $\displaystyle\int_0^2 \frac{dx}{\sqrt{(x+2)(x^2+3x+2)}}$

4. Find the arclength of the curve $y = x^2/2,\ 0 \le x \le 1$.

5. A right circular cone is circumscribed about a sphere of radius 1. Find the dimensions of the cone of minimum volume. Be sure to justify your answer.

6. A fuel tank is in the shape of a right circular cylinder with base radius a and height L, positioned on its side. If fuel is being pumped into the tank at the rate of q units^3/min, at what rate is the fuel level rising when it's at depth $a/2$?

7. Determine the volume of the solid generated by rotating the triangle with vertices $(4,0)$, $(6,2)$, and $(6,-2)$ about the y-axis.

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8. Find the value of x for which the integral

$$\int_x^{2x} e^{-t^2}\,dt$$

is greatest. (Hint: No one knows an explicit antiderivative for e^{-t^2}.)

9. Alice and Betty are at opposite ends of a single trail on a mountain. They start hiking at precisely noon. At precisely 6 pm, Betty is at Alice's starting point and Alice is at Betty's starting point. Prove that there is some time after noon and before 6 pm at which their walking speeds are equal.

10. At the edge of a circular arena of radius a there is a light L, as pictured. Bert runs from B (one fourth the circumference around from L) at 10 ft/sec towards the center O. At what rate is his shadow moving along the far side of the arena when he is halfway to the center? Hint: Let x, θ, s be as marked in the diagram.