Study Guide for Real Analysis Exam

Undergraduate Analysis

Continuity and differentiation in one real variable
Metric spaces and compactness in analysis
Sequences and series
Uniform convergence and uniform continuity
Taylor’s theorem
Weierstrass approximation theorem

References: [2] Chapters 2, 3, 4, 5, 7; [1] Sections 0.6.

Measure and Integration

Measures on $\mathbb{R}^n$ and on $\sigma$-algebras
Measurable and integrable functions
Convergence theorems: Fatou’s lemma, the monotone and dominated convergence theorems and Egoroff’s theorem
Notions of convergence: uniform, pointwise, almost everywhere, and in norm
Fubini and Tonelli theorems

References: [1] Chapters 1, 2; [3] Chapters 1, 2, 6.

Function Spaces

The Banach spaces $L^1$ and $L^\infty$:
Completeness
Convolutions and approximations to the identity
Linear functionals and realizing $L^\infty$ as the dual of $L^1$
Hilbert space and $L^2$ spaces:
Schwarz inequality and orthogonality
Linear functionals and the Riesz representation theorem
Bessel’s inequality, orthonormal basis, and Parseval’s identity
Trigonometric series: trigonometric polynomials are dense in both $C([0,1])$ (with respect to the uniform metric) and in $L^2([0,1])$


References


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