Top

Preliminary Exam in Topology
Wednesday, May 11, 1994, 1:00–4:00 pm

1. Let $A_1, A_2, A_3, \ldots$ be a sequence of connected subsets of the topological space X, such that $A_n \cap A_{n+1} \neq \emptyset$ for all n. Prove that $\bigcup_{n=1}^{\infty} A_n$ is connected.

2. Let $f : X \to S$ be a surjective function from the topological space X to the set S. Prove that the quotient topology on S induced by f is the finest topology relative to which f is continuous.

3. Let X be a compact metric space with metric d, and let $f : X \to X$ be a function such that $d(f(x), f(y)) = d(x, y)$ for all x, $y \in X$. Prove that f is continuous and surjective.

4. Classify compact connected surfaces S with Euler characteristic $\chi(S) \geq -2$. In other words, give a list of surfaces so that every compact connected surface S with $\chi(S) \geq -2$ is homeomorphic to a surface on the list, and no two surfaces on the list are homeomorphic. (The surfaces can be orientable or nonorientable, and they can have empty boundary or nonempty boundary.)

5. Show how to use van Kampen's theorem to compute the fundamental group of a compact, connected, orientable surface of genus g.

6. Describe the universal cover of $X = \mathbb{R}P^2 \vee S^1$, the one-point union of the real projective plane and the circle. Compute the fundamental group of X, and show how it acts on the universal cover as the group of deck transformations.

7. Describe a finite CW-complex structure on $\mathbb{C}P^2 \times S^1$, the product of the complex projective plane and the circle. Compute the cellular chain complex and the cellular homology.

8. Let $f : S^n \to S^n$ be a continuous map, with n even ($n \geq 0$). Prove that either there exists $x \in S^n$ such that $f(x) = -x$, or there exists $x \in S^n$ such that $f(x) = x$. For each odd $n \geq 1$, define a continuous map $f : S^n \to S^n$ such that for all $x \in S^n$, $f(x) \neq -x$ and $f(x) \neq x$.

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