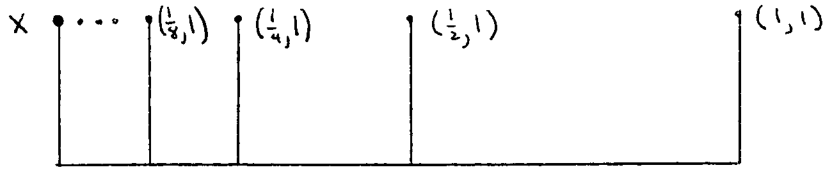
Topology Prelim

March 30, 1998

Directions: Do all of the problems. Each problem is worth 10 points.

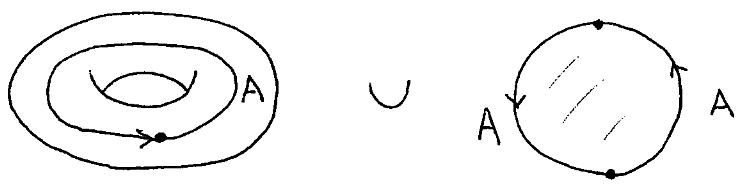
1. Prove or disprove: The space drawn below can be contracted to a point in such a way that the point x is fixed throughout the homotopy.



2. Define an equivalence relation on $\mathbb{R}^2$ by defining $(x, y) \sim (z, w)$ if $xy = zw$. Is the quotient space Hausdorff?

3. Prove that a retract of a Hausdorff space is closed.

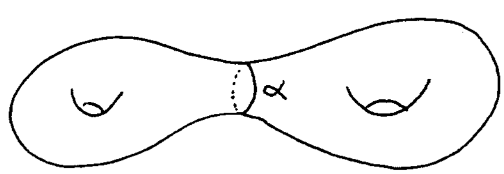
4. Let X be the union of a torus and a disk as shown below. Compute $H_*(X)$.



5. State and prove the unique path lifting theorem for covering spaces.

6. Compute $H_0(GL(2, \mathbb{R}))$ where $GL(2, \mathbb{R})$ denotes the group of 2×2 invertible matrices.

7. Let α be an embedded circle in a surface, S, of genus 2 such that α separates S into two components, each of which is homeomorphic to a punctured torus. Prove that α is not null-homotopic.



8) Find a presentation for $\pi_1(S^3 - k)$ where k is the trefoil knot drawn below.