UNIVERSITY OF GEORGIA

DEPARTMENT OF MATHEMATICS
MATH 2250 - FINAL EXAM
SPRING 2026

PRINTED FIRST & LAST NAME

SOLUTIONS

UGAID – YOUR 81#

INSTRUCTOR

CLASS TIME

INSTRUCTIONS

This exam has **20** questions and is out of **150** points.

- The exam lasts 3 hours and it has two parts: the first one consists of Multiple Choice (MC) questions, and the second of Free Response (FR) ones. You must show work for both parts, unless explicitly told otherwise. An unjustified answer will receive no credit. If you are using a shortcut, explain it.
- Your work must be neat and organized. For multiple-choice questions, **bubble in** your answer, and for free-response questions, enter your answer in the provided box. There is only one correct choice for each MC question.
- Smart devices, including smartwatches and cell phones, are prohibited and must not be within reach.
- If you plan to use a calculator, only TI-30XS MultiView (the name must match exactly) is permitted; no other calculators or sharing of calculators is allowed.
- Provide an exact answer for each problem. Answers containing symbolic expressions such as $\cos(3)$ and $\ln(2)$ are perfectly acceptable, but please simplify or evaluate when possible, for example, $\sin(\pi/2) = 1$.
- If additional space is needed, use the last two pages. Write “cont’d” (continued) in the designated area and continue on the scrap paper by first writing the problem number and then continuing your solution. Work outside the specified area without any indication will not be graded.

SCRAP PAPER

Do NOT tear this page off!

Part I: Multiple Choice

Show work and **bubble in** the final answer.

1. [5 pts] A particle moves along a line with position $s(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and $s(t)$ in meters. Determine the velocity of the particle at $t = 1$ second.

(A) -3 m/s (B) 0 m/s (C) 3 m/s (D) 4 m/s (E) 9 m/s

$$s'(t) = 3t^2 - 12t + 9$$

$$s'(1) = 3 \cdot 1^2 - 12 \cdot 1 + 9$$

$$= 0$$

$$\boxed{s'(1) = v(1) = 0 \ m/s}$$

2. [5 pts] Find the tangent line to the parabola $y = x^2 + 2x$ at $x = 1$.

(A) $y = 3x + 1$ (B) $y = 4x + 1$ (C) $y = 2x + 1$ (D) $y = 3x - 1$ (E) $y = 4x - 1$

$$m_{tan}(1) = y'(1) = 2x + 2 \Big|_{x=1} = 2 \cdot 1 + 2 = 4$$

Equation of the tangent line at $x = 1$, $y = 1^2 + 2 \cdot 1 = 3$ is:

$$y - 3 = 4(x - 1)$$

$$\Updownarrow$$

$$\boxed{y = 4x - 1}$$

3. [5 pts] The function

$$y = e^{x^2-6x}$$

has a critical point at $x = 3$. Determine the nature of this extremum.

- (A) local minimum
- (B) local maximum
- (C) point of inflection
- (D) neither a local minimum nor a local maximum
- (E) there is not enough information to answer this question

$y' = (2x-6)\underbrace{e^{x^2-6x}}_{>0}$

Indeed $y'=0$ at $x=3$.

Sign of f': $-$ $\overset{3}{\circ}$ $+$

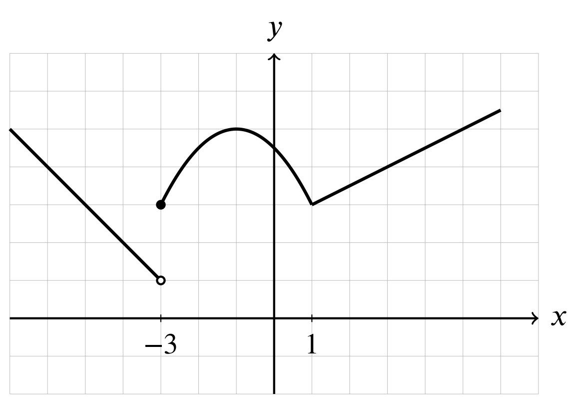
local min.

4. [5 pts] The graph of a function $f(x)$ is given below. Which of the following statements are true? **No need to show work.**

I. $f(x)$ has a jump discontinuity at $x = -3$. ✓

II. $f(x)$ is differentiable at $x = -3$. ✗

III. $f(x)$ has a local minimum at $x = -3$. ✗



(A) I only (B) III only (C) I and II only (D) I and III only (E) II and III only

5. [5 pts] Find the slope of the tangent line to the curve

$$e^x + \arctan(y) - xy^2 = e \qquad (*)$$

at the point $(1, 0)$.

(A) $-e^2$ (B) $-e$ (C) -1 (D) 0 (E) e

Differentiating both sides of $(*)$ with respect to x gives us:

$$e^x + \frac{1}{1+y^2} \cdot y' - y^2 - 2xyy' = 0$$

At $(1,0)$:

$$e + \frac{1}{1+0^2} \cdot y' - 0^2 - 2 \cdot 1 \cdot 0\, y' = 0$$

$$\boxed{y' = -e}$$

6. [5 pts] **No need to show work.** In this article from 1979 [1],

The New York Times

Money-Supply Growth Is Continuing to Slow

By John H. Allan
Jan. 26, 1979

Growth of the nation's money supply is continuing to slow, according to figures released by the Federal Reserve yesterday.

the author describes the money supply in the United States. Given the information provided, at the beginning of 1979, the graph of the nation's money supply would be

(A) decreasing, concave down. (B) decreasing, concave up. (C) increasing, concave down.
(D) increasing, concave up. (E) none of the above.

[1] Retrieved on April 15, 2026 from `https://www.nytimes.com`

7. [5 pts] Find the slope of the line that is tangent to $y = x^{x+2}$ at $(1, 1)$.

(A) 0 (B) 1 (C) 2 (D) e (E) 3

$$y = x^{x+2} = e^{\ln x^{x+2}} = e^{(x+2)\ln x}$$

$$y' = e^{(x+2)\ln x}\left((x+2)\cdot\frac{1}{x} + \ln x\right)$$

$$m_{tan}\Big|_{(1,1)} = y'\Big|_{(1,1)} = e^{(1+2)\underbrace{\ln 1}_{=0}}\left((1+2)\cdot\frac{1}{1} + \underbrace{\ln 1}_{=0}\right) = 3$$

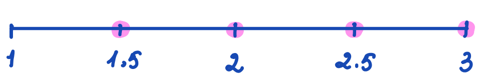
$$\boxed{m_{tan}\Big|_{(1,1)} = y'\Big|_{(1,1)} = 3}$$

8. [5 pts] Using the right endpoint rule with 4 rectangles, approximate the integral $\displaystyle\int_1^3 \frac{\cos x}{x}\,dx$.

The following table provides values of $\dfrac{\cos x}{x}$, correct to the nearest tenth:

x	1	1.5	2	2.5	3
$\frac{\cos x}{x}$	0.5	0.0	−0.2	−0.3	−0.3

(A) −0.4 (B) −0.3 (C) −0.2 (D) 0.0 (E) 0.2



$$\Delta x = \frac{3-1}{4} = 0.5$$

$$\int_1^3 \overset{= f(x)}{\frac{\cos x}{x}}\,dx \approx \left(f(1.5) + f(2) + f(2.5) + f(3)\right)\cdot \Delta x$$

$$= \left(0 + (-0.2) + (-0.3) + (-0.3)\right)\cdot 0.5$$

$$= (-0.8)(0.5)$$

$$= -0.4$$

$$\boxed{\int_1^3 \frac{\cos x}{x}\,dx \approx -0.4}$$

9. [5 pts] Evaluate

$$\int_1^2 \frac{1}{3x}\,dx.$$

(A) $-\frac{1}{6}$ (B) $\frac{1}{3}\ln(2)$ (C) $\ln(6)-\ln(3)$ (D) $\frac{1}{3}\ln(3)$ (E) $\ln\left(\frac{2}{3}\right)$

$$\int_1^2 \frac{1}{3x}\,dx$$

$$= \frac{1}{3}\int_1^2 \frac{1}{x}\,dx$$

$$= \frac{1}{3}\ln|x|\Big|_1^2$$

$$= \frac{1}{3}(\ln 2 - \ln 1)$$

$$= \frac{1}{3}\ln 2 \quad //$$

So, $\boxed{\int_1^2 \frac{1}{3x}\,dx = \frac{1}{3}\ln 2}$

10. [5 pts] Evaluate

$$\int_0^{\frac{\pi}{3}} \tan(x)\sec^2 x\,dx.$$

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\sqrt{3}$ (D) 1 (E) $\ln(3)$

$$\int_0^{\pi/3} \tan x \sec^2 x\,dx$$

- let $u = \tan x$, $du = \sec^2 x\,dx$ or $u = \sec\theta$, $du = \sec\theta\tan\theta\,d\theta$
- $x = 0 .. \pi/3$, $u = 0 .. \sqrt{3}$

$$= \int_0^{\sqrt{3}} u\,du$$

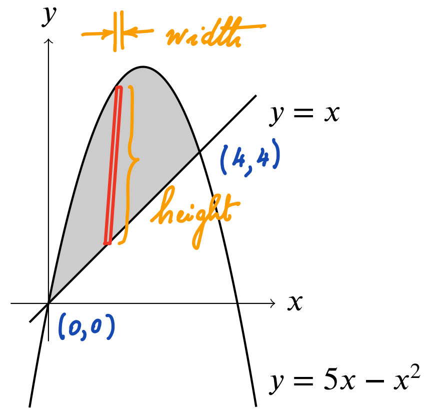
$$= \frac{u^2}{2}\Big|_0^{\sqrt{3}}$$

$$= \frac{3}{2} - 0$$

$$= \frac{3}{2}$$

So, $\boxed{\int_0^{\pi/3} \tan x \sec^2 x\,dx = \frac{3}{2}}$

11. [5 pts] Find the area of the shaded region.



Ⓐ $\frac{16}{3}$ Ⓑ 8 Ⓒ $\frac{32}{3}$ Ⓓ $\frac{64}{3}$ Ⓔ 16

• Points of intersection:

$5x - x^2 = x \iff x^2 - 4x = 0 \Rightarrow x = 0$ and $x = 4$.

• $\text{Area} = \int_0^4 [(5x - x^2) - x]\, dx$

$= \int_0^4 (4x - x^2)\, dx$

$= \left(\frac{4x^2}{2} - \frac{x^3}{3}\right)\Big|_0^4$

$= \left(2 \cdot 4^2 - \frac{4^3}{3}\right) - 0$

$= 32 - \frac{64}{3}$

$= \frac{32}{3}$

$\boxed{\text{Area} = \frac{32}{3}}$

12. [5 pts] Find $F'\left(\frac{\pi}{3}\right)$ if

$$F(x) = \int_{\sqrt{x}}^{1} \frac{2\pi\cos(t^2)}{t}\, dt.$$

Ⓐ $-\frac{3\sqrt{3}}{2}$ Ⓑ $-\frac{3}{2}$ Ⓒ $-\frac{3\sqrt{3}}{4}$ Ⓓ $\frac{\sqrt{3}}{2}$ Ⓔ $\frac{3}{2}$

$F(x) = -\int_1^{\sqrt{x}} \frac{2\pi\cos(t^2)}{t}\, dt$

$F'(x) = -\frac{\cancel{2}\pi\cos((\sqrt{x})^2)}{\sqrt{x}} \cdot \frac{1}{\cancel{2}\sqrt{x}}$

$= -\frac{\pi\cos x}{x}$

$F'\left(\frac{\pi}{3}\right) = -\frac{\cancel{\pi}\cos\pi/3}{\cancel{\pi}/3} = -\frac{3}{2}$

$\boxed{F'\left(\frac{\pi}{3}\right) = -\frac{3}{2}}$

Part II: Free Response

Unless specified otherwise, show all work leading to your final answer clearly and in a structured manner.

13. [15 pts] Determine the following limits

(5 PTS) (a) $\lim_{x\to 2} \sqrt{3x^3+1}$

$= \sqrt{3 \cdot 2^3 + 1}$

$= \sqrt{25}$

$= 5$ //

ANSWER

5

(5 PTS) (b) $\lim_{t\to 0} \frac{\sqrt{1+t}-1}{t}$

$\left[\frac{0}{0}\right]$

$= \lim_{t\to 0} \frac{\sqrt{1+t}-1}{t} \cdot \frac{\sqrt{1+t}+1}{\sqrt{1+t}+1}$

$= \lim_{t\to 0} \frac{\cancel{1}+t-\cancel{1}}{t(\sqrt{1+t}+1)}$

$= \lim_{t\to 0} \frac{\cancel{t}}{\cancel{t}(\sqrt{1+t}+1)}$

$= \frac{1}{2}$ //

ANSWER

$\frac{1}{2}$

(5 PTS) (c) $\lim_{x\to+\infty} \sqrt{x^2+10x} - \sqrt{x^2+4x}$

$[\infty - \infty]$

$$= \lim_{x\to\infty} \left(\sqrt{x^2+10x} - \sqrt{x^2+4x}\right) \cdot \frac{\left(\sqrt{x^2+10x} + \sqrt{x^2+4x}\right)}{\left(\sqrt{x^2+10x} + \sqrt{x^2+4x}\right)}$$

$$= \lim_{x\to\infty} \frac{(\cancel{x^2}+10x) - (\cancel{x^2}+4x)}{\sqrt{x^2+10x} + \sqrt{x^2+4x}}$$

$$= \lim_{x\to\infty} \frac{6x}{\sqrt{x^2+10x} + \sqrt{x^2+4x}}$$

$$= \lim_{x\to\infty} \frac{6x}{x+x}$$

(using asymptotic behaviour or L'Hopital or factoring out the leading terms.)

$$= 3$$

ANSWER

3

14. [15 pts] For each function below, answer the corresponding question.

(5 PTS) (a) Let $f(x) = \dfrac{x+9}{x^2+3}$. Find $f'(1)$.

Quotient Rule

$$f'(x) = \frac{(x^2+3)\cdot 1 - (x+9)\cdot 2x}{(x^2+3)^2}$$

$$= \frac{x^2+3-2x^2-18x}{(x^2+3)^2}$$

$$= \frac{-x^2-18x+3}{(x^2+3)^2}$$

$$f'(1) = \frac{-1^2-18\cdot 1+3}{(1^2+3)^2} = -\frac{16}{16} = -1$$

ANSWER

$f'(1) = -1$

(5 PTS) (b) If $\dfrac{dg}{dx} = (x^2 - 2x - 2)e^x$, find the x-coordinates of all inflection points of $g(x)$, if any.

$$\frac{d^2g}{dx^2} = (2x-2)e^x + (x^2-2x-2)e^x$$

$$= (x^2 - \cancel{2x} - 2 + \cancel{2x} - 2)e^x$$

$$= (x^2-4)\underbrace{e^x}_{>0}$$

$\dfrac{d^2g}{dx^2} = 0$ when $x^2 - 4 = 0$ i.e. $x = \pm 2$

		-2		2	
sign of g''	$+$	0	$-$	0	$+$

ANSWER

$x = -2$ and $x = 2$

(5 PTS) (c) Let $h(x) = \left(\ln(x^2+1)\right)^5$. Find all values of x where the tangent line to $h(x)$ is horizontal.

$$h'(x) = 5\left(\ln(x^2+1)\right)^4 \cdot \frac{2x}{x^2+1}$$

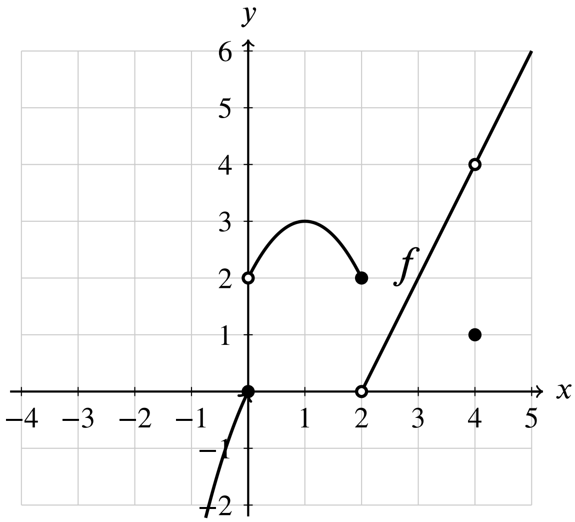
$$= \frac{10x}{x^2+1} \underbrace{\left(\ln \underbrace{(x^2+1)}_{>1}\right)^4}_{>0}$$

Tangent line is horizontal when $h'(x) = 0$, i.e. $\dfrac{10x}{x^2+1} = 0 \Rightarrow x = 0.$ //

ANSWER

$x = 0$

15. [15 pts] For this problem, the graph of $f(x)$ is given below. Note that the graph extends towards positive or negative infinity.



No need to show your work for parts (a), (b), and (c).

(3 PTS) (a) $\lim_{x\to 0^+} f(x) = 2$

ANSWER

2

(3 PTS) (b) $\lim_{x\to 4}(3f(x)-2) = 3\underbrace{\lim_{x\to 4} f(x)}_{=4} - 2 = 10$

ANSWER

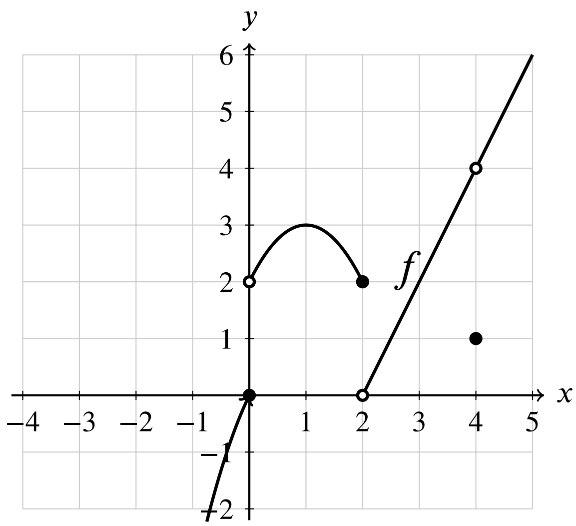
10

(3 PTS) (c) $f'(1) = 0$

ANSWER

0

The graph of $f(x)$ from the previous page is shown again below. Note that the graph extends towards positive or negative infinity.



(3 PTS) (d) List the x-coordinates of all critical points of $f(x)$. **No need to show your work.**

ANSWER

$x = 0, 1, 2, 4.$

(3 PTS) (e) $\int_{2.5}^{3.5} f(x)\,dx$ = Area of trapezoid with height 1, little base $\underbrace{1}_{= f(2.5)}$ and big base $\underbrace{3}_{= f(3.5)}$

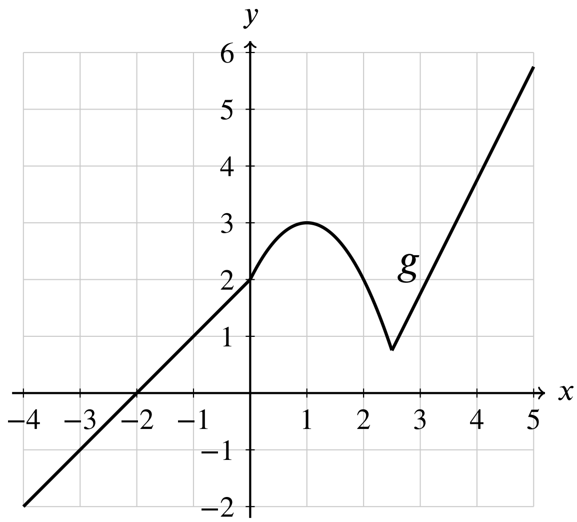
$= \frac{1}{2} \cdot 1 \cdot (1 + 3)$

$= 2$

ANSWER

2

16. [8 pts] The graph of $g(x)$ is given below. Note that the graph extends towards positive or negative infinity.



(4 PTS) (a) We know that $g'(2) = -2$. Using linearization, approximate $g(1.95)$.

$$g(1.95) \approx L(1.95) = g(\overset{''a}{2}) + g'(\overset{''a}{2})(\overset{''x}{1.95} - \overset{''a}{2})$$

$$= 2 + (-2)(-0.05)$$

$$= 2 + \cancel{2}\cdot\frac{\cancel{5}\ 1}{\cancel{100}\ \cancel{20}\ 10}$$

$$= 2.1$$

ANSWER

$g(1.95) \approx 2.1$

(4 PTS) (b) For which value of a is $\int_a^4 g(x)\,dx$ maximal? Justify.

To make $\int_a^4 g(x)\,dx = F(a)$ maximal, we want to include as much positive area as possible and avoid negative area. $g(x) < 0$ for $x \in (-\infty, -2)$ and $g(x) > 0$ for $x \in (-2, \infty)$ so the maximum occurs at $a = -2$.

ANSWER

$a = -2$

17. [6 pts] A nuclear scientist studies a particle moving around a circle of radius 5 m around the origin:

$$x^2 + y^2 = 25.$$

As the particle passes through the point of coordinates $(3, 4)$, its x position is changing at a rate of $\left.\frac{dx}{dt}\right|_{x=3,y=4} = 2$ m/s. How fast is y changing at that moment?

Relation :

$$x^2 + y^2 = 25$$

Differentiating both sides with respect to t:

$$\cancel{2}x \frac{dx}{dt} + \cancel{2}y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

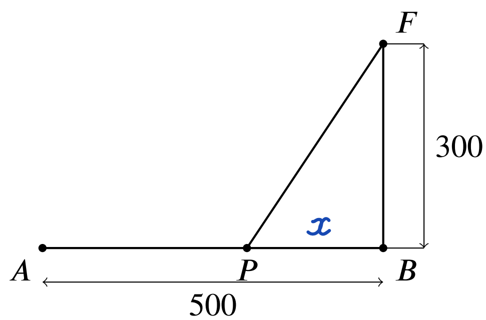
At $(3,4)$:

$$\left.\frac{dy}{dt}\right|_{(3,4)} = -\frac{3}{4}\cdot 2 = -\frac{3}{2}$$

As the particle passes through the point $(3,4)$, the particle's y-coordinate is changing at a rate of $\boxed{-3/2 \text{ m/s}}$.

18. [10 pts] A factory F produces textiles that must be delivered to a town A located along a straight railway line. The railway segment from A to a point B has length 500 mi. The factory is situated off the railway, at a distance of 300 mi from B, along a line perpendicular to the track (see figure below).

To organize transportation, a road will be built from the factory to some point P on the railway. Goods will first be carried by truck from F to P, and then transported by train from P to A. The cost per unit distance of transport by truck is \$2 per mile, while the cost of transport by train is \$1 per mile.
The aim is to determine the position of the point P on the railway that minimizes the total transportation cost.



Introduce the variable $x = PB$.

(5 PTS) (a) Express in terms of x:

Quantity	Answer
distance PA	$500 - x$
distance PF	$\sqrt{300^2 + x^2}$
Cost of the train leg	$1 \cdot (500 - x)$
Cost of the truck leg	$2\sqrt{300^2 + x^2}$
Total cost of the transport	$500 - x + 2\sqrt{300^2 + x^2}$

(Problem 18 continues on the next page)

(5 PTS) (b) Solve the problem using calculus and justify your finding. Give your final answer as the distance between A and P.

$$C(x) = 500 - x + 2\sqrt{300^2 + x^2}, \quad x \in (0, 500)$$

$$C'(x) = -1 + \cancel{2} \cdot \frac{1}{\cancel{2}} (300^2 + x^2)^{-1/2} \cdot 2x$$

$$= -1 + \frac{2x}{\sqrt{300^2 + x^2}}$$

$$C'(x) = 0 \text{ iff } \frac{2x}{\sqrt{300^2+x^2}} = 1 \iff 2x = \sqrt{300^2 + x^2}$$

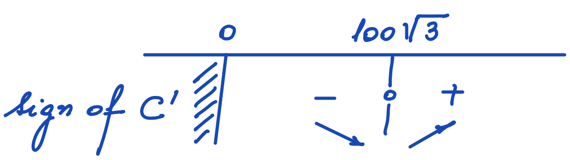
$$\iff 4x^2 = 300^2 + x^2$$

$$\iff 3x^2 = 90000$$

$$\iff x^2 = 30000$$

$$\iff x = \pm 100\sqrt{3}$$

outside of the domain so $x = 100\sqrt{3}$ is the only critical value



$C(x)$ has a minimum at $x = 100\sqrt{3}$ and $x = 100\sqrt{3}$ is the only critical value on $(0, 500)$, so it has an absolute minimum at $x = 100\sqrt{3}$.

So the optimal point P is located $500 - 100\sqrt{3}$ mi from A along the railroad.

ANSWER

$\boxed{500 - 100\sqrt{3} \text{ mi}}$

19. [13 pts] Consider the function $f(x) = \dfrac{e^x}{x-1}$.

(5 PTS) (a) Determine all points where $f(x)$ is discontinuous, and use limits to classify each discontinuity as removable, a jump, a vertical asymptote, or otherwise.

$f(x) = \dfrac{e^x}{x-1}$ is discontinuous at $x=1$.

$$\lim_{x\to 1^-} \frac{\overbrace{e^x}^{+}}{\underbrace{x-1}_{-}} \overset{\left[\frac{1}{0}\right]}{=} -\infty$$

$$\lim_{x\to 1^+} \frac{\overbrace{e^x}^{+}}{\underbrace{x-1}_{+}} \overset{\left[\frac{1}{0}\right]}{=} +\infty$$

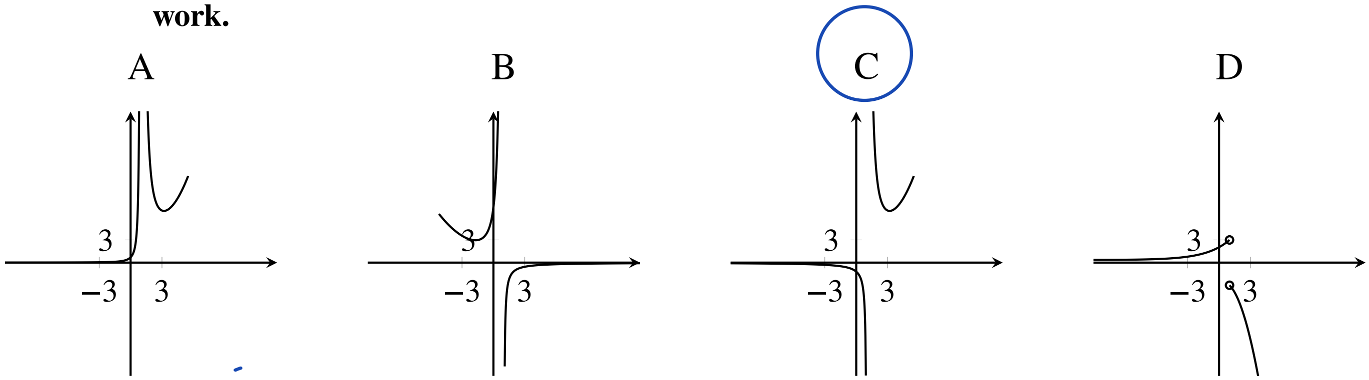
Thus, $f(x)$ has a vertical asymptote at $\boxed{x=1}$.

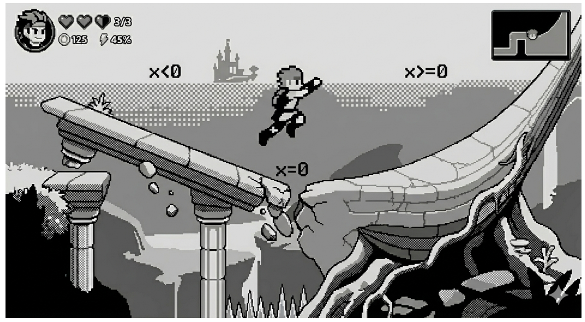
(5 PTS) (b) Determine all horizontal asymptotes of $f(x)$, if any. Justify your answer.

$\lim_{x\to -\infty} \dfrac{e^x}{x-1} = 0 \quad \therefore \boxed{y=0}$ is a horizontal asymptote for $f(x)$.

$\lim_{x\to +\infty} \dfrac{e^x}{x-1} = \infty \quad \therefore$ there is no horizontal asymptote as $x \to +\infty$.

(3 PTS) (c) Which of the following is the graph of $f(x)$? Circle the correct graph. **No need to show work.**





Dr. Calculus will save you! (Image generated by Gemini)

20. [8 pts] In the video game Dr. Calculus, the hero is moving on two moving platforms which can be described by a family of piecewise functions depending on a parameter k (time):

$$p_k(x) = \begin{cases} \sin(k)x + \cos(k) & \text{when } x < 0 \\ x^2 - \frac{\sqrt{2}}{2}x + \sin(k) & \text{when } x \geq 0 \end{cases}$$

(4 PTS) (a) Dr. Calculus has problems with his knees! Help him find the value(s) of the parameter k in $[0, 2\pi]$ where the two platforms connect. (i.e. p_k is continuous)

$\sin k\, x + \cos k$ and $x^2 - \frac{\sqrt{2}}{2}x + \sin k$ are continuous for all k and x.

For p_k to be continuous at $x=0$, where the two platforms connect:

$$\lim_{x\to 0^-} (\sin k\, x + \cos k) = \lim_{x\to 0^+} \left(x^2 - \frac{\sqrt{2}}{2}x + \sin k\right) = p_k(0)$$

$\cos k = \sin k$ i.e. on $[0, 2\pi]$, $k = \frac{\pi}{4}, \frac{5\pi}{4}$

ANSWER

$k = \frac{\pi}{4}$ and $k = \frac{5\pi}{4}$

(4 PTS) (b) For which value(s) of k do the platforms transition smoothly? (i.e. is p_k differentiable)

The platforms transition smoothly if p_k does not have a discontinuity, a vertical drop or a sharp corner at $x=0$ (outside of $x=0$ the platforms are smooth, they are described by polynomials). Continuity is already acquired so we only need:

$$\lim_{x\to 0^-} \overbrace{\sin k}^{p_k',\ x<0} = \lim_{x\to 0^+} \overbrace{2x - \frac{\sqrt{2}}{2}}^{p_k',\ x\geq 0}$$

$$\sin k = -\frac{\sqrt{2}}{2} \Rightarrow \boxed{k = \frac{5\pi}{4}}.$$

ANSWER

$k = \frac{5\pi}{4}$

SCRAP PAPER

Do NOT tear this page off!