Show your work and justify all your reasoning.

1. (a) Give the power series expansion about \( z = 1 \) of the branch of \( f(z) = z^i \) with \( f(1) = 1 \). Find its radius of convergence.
(b) Find all solutions of the equation \( z^i = i \).

... 2. Let \( U \subset \mathbb{C} \) be a simply connected domain and \( f : U \to \mathbb{C} \) a holomorphic function. Show, using only advanced calculus, that
\[
\int_{\gamma} f(z) \, dz = 0
\]
for any smooth closed loop \( \gamma \subset U \).

3. Prove that there is no meromorphic function \( f \) on \( \mathbb{C} \) such that \( f(x) = \arctan x \) for all \( x \in \mathbb{R} \).

4. Give the Laurent expansion of \( \frac{1}{z(z-1)} \) in (a) the annulus \( \{0 < |z| < 1\} \) and (b) the annulus \( \{1 < |z| < 2\} \).

5. Compute
\[
\int_0^\infty \frac{\sqrt{x}}{x^2 + 1} \, dx.
\]

6. Give a formula for a conformal mapping from the region
\[
U := \{re^{i\theta} : 0 < r < 1, \, 0 < \theta < \frac{\pi}{4}\}
\]
on onto the unit disk.

7. Suppose \( f(z) \) is a meromorphic function on \( \mathbb{C} \) such that \( \lim_{z \to \infty} |f(z)| \) exists (possibly taking the value \( \infty \)). Show that \( f \) is a rational function.

8. Let \( U \subset \mathbb{C} \) be a domain and \( F : [0,1] \times U \to \mathbb{C} \) a bounded continuous function such that \( z \mapsto F(t,z) \) is holomorphic on \( U \) for every \( t \in [0,1] \). Define \( f : U \to \mathbb{C} \) by
\[
f(z) = \int_0^1 F(t,z) \, dt.
\]
Prove that \( f \) is holomorphic.