1. a) State the Riemann Mapping Theorem.
   b) Give an explicit conformal mapping which takes \( \mathbb{C} \setminus [(-\infty,-1) \cup (1,\infty)] \)
   onto the unit disc.
   c) Is the punctured disc \( \mathbb{C} \setminus \{0\} \) conformally equivalent to any
   annulus \( \{ z \in \mathbb{C} : a < |z| < b \} \) with \( a, b \) finite?
   Justify your answer.

2.) Let \( \Gamma \) be the square with corners \(-i, +i, -2+i, -2-i\), traversed
   counterclockwise; let \( f(z) = 2/(z^3-1) \). Compute the winding
   number of the curve \( f(\Gamma) \) about the point \( w = -1 \), giving
   \( f(\Gamma) \) the orientation it inherits from \( \Gamma \).

3.) Calculate \( \int_0^{\infty} \frac{\cos(x)}{1 + x^2} \, dx \).

4.) Compute the Taylor expansion, about the origin, of
   \( f(z) = \arctan(z^2) \), and determine its radius of convergence.

5.) The Gamma function is defined for \( \text{Re}(z) > 0 \) by
   \( \Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} \, dt \).
   a) Show that \( \Gamma(z) \) extends to a meromorphic function on the whole
      complex plane.
   b) Determine the locations of the poles of \( \Gamma(z) \), their orders,
      and their residues.

6.) Describe the Riemann surface of the algebraic function
   \( f(z) = \sqrt[3]{z^3 - z} \), including branch points and the behavior at \( \infty \).

7.) Construct a function analytic on the unit disc, which has a zero of
   order \( n \) at \( z_n = 1 - 1/n \), for \( n = 1, 2, 3, \ldots \).

8.) Let \( f(z) \) be an entire function with the property that
   \( |f(z)| \leq |f(z^2)| \) for all \( z \). Show that \( f(z) \) must be constant.