



UNIVERSITY OF
GEORGIA

DEPARTMENT OF MATHEMATICS
MATH 2250 - FINAL EXAM
SPRING 2025

PRINTED NAME : _____

STUDENT ID : _____

DATE : ____/____/____

GRADE
150

INSTRUCTOR : _____

CLASS TIME : _____

INSTRUCTIONS

Nº	SCORE	MAX
1		5
2		5
3		5
4		5
5		5
6		5
7		5
8		5
9		5
10		5
11		5
12		5
13		5
14		5
15		5
16		5
17		5
18		5
19		16
20		10
21		24
22		10
TOTAL		150

- The exam lasts 3 hours and it has two parts: the first one consists of Multiple Choice (MC) questions, and the second part of Free Response (FR) ones. You must show work for both parts, unless explicitly told otherwise. An unjustified answer will receive no credit. If you are using a shortcut, explain it.
- Your work must be neat and organized. Circle the answer for MC questions and put a box around the final answer for the FR questions. There is only one correct choice for each MC question.
- Smart devices, including smart watches and cell phones, are prohibited and must not be within reach.
- If you plan to use a calculator, only TI-30XS MultiView (the name must match exactly) is permitted; no other calculators or sharing of calculators is allowed.
- Provide an exact answer for each problem. Answers containing symbolic expressions such as $\cos(3)$ and $\ln(2)$ are perfectly acceptable, but $\sin(\pi/2) = 1$.
- If additional space is needed, use the last two pages. Write “cont’d” (continued) in the designated area and continue on the scrap paper by first writing the problem number and then continuing your solution. Work outside the specified area without any indication, will not be graded.

SCRAP PAPER

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Part I: Multiple Choice

Show work and circle the final answer.

1. [5 pts] If $y = 2x^3 + 3x^2 - 12x - 10$, find $y'(-1)$.

- (A) -18
- (B) -12
- (C) -6
- (D) 0
- (E) 6

PTS

2. [5 pts] If $f(\theta) = \sin \theta$, find $f^{(11)}(\pi)$, the 11th derivative of $f(\theta)$ at $\theta = \pi$.

- (A) $-\pi$
- (B) -1
- (C) 0
- (D) 1
- (E) π

PTS

3. [5 pts] How many critical points of $y = x^5 - \frac{20}{3}x^3$ are NOT local extrema?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 5

PTS

4. [5 pts] If $y(x) = \sqrt{1-x^2} + x \arcsin x$, compute $y'(x)$.

- (A) $\frac{x}{\sqrt{1-x^2}}$
- (B) $\frac{1}{\sqrt{1-x^2}}$
- (C) $\arcsin x - \frac{2x}{\sqrt{1-x^2}}$
- (D) $\arcsin x - \frac{x}{\sqrt{1-x^2}}$
- (E) $\arcsin x$

PTS

5. [5 pts] A person walks away from a lamp post at a rate of 3 ft/sec. The lamp post is 10 ft tall, and the person is 5 ft tall. How fast is the length of the person's shadow increasing when the person is 15 ft from the lamp post?

- (A) $\frac{3}{2}$ ft/sec
- (B) $\frac{10}{3}$ ft/sec
- (C) $\frac{5}{3}$ ft/sec
- (D) 2 ft/sec
- (E) 3 ft/sec

_____ PTS

6. [5 pts] What is the slope of the tangent line to $xy^2 + e^y = x^2$ at the point $(1, 0)$?

- (A) -1
- (B) 0
- (C) $\frac{1}{2}$
- (D) 1
- (E) 2

_____ PTS

7. [5 pts] Consider the function

$$f(x) = \begin{cases} x^2 + ax - 3x + 3 - a, & \text{if } x \leq 1 \\ \frac{2x}{x^2 + 1}, & \text{if } x > 1 \end{cases}$$

What should a be so that the function is differentiable everywhere?

- Ⓐ $a = 0$
- Ⓑ $a = \frac{1}{2}$
- Ⓒ $a = \frac{1}{3}$
- Ⓓ $a = \frac{1}{4}$
- Ⓔ $a = 1$

_____ PTS

8. [5 pts] If $f(x) = x^3 + 8x + \cos(3x)$ and $g(x) = f^{-1}(x)$, find the slope of the tangent line to $g(x)$ at the point $(1, 0)$.

- Ⓐ $\frac{1}{11}$
- Ⓑ $\frac{1}{8}$
- Ⓒ $-\frac{1}{8}$
- Ⓓ $\frac{1}{11 + 3 \sin 3}$
- Ⓔ 8

_____ PTS

9. [5 pts] Let f and g be differentiable functions, and suppose the following values are known:

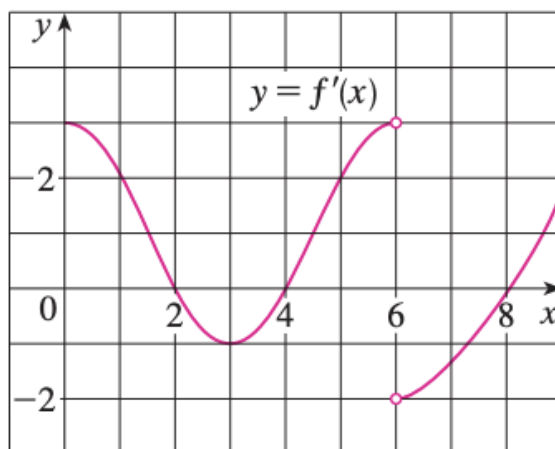
$$\begin{array}{llll} f(a) = -4, & f'(a) = 8, & g(a) = c, & g'(a) = b, \\ f(c) = 3, & f'(c) = 2, & g(c) = 5, & g'(c) = 1. \end{array}$$

Define $h(x) = [f(g(x))]^2$. What is the value of $h'(a)$?

- (A) $4b$
- (B) $6b$
- (C) $12b$
- (D) $15b$
- (E) $24b$
- (F) $36b$

PTS

10. [5 pts] Below is the graph of the derivative f' of a continuous function f .



At what value(s) of x does f have a local maximum?

- (A) 0 only
- (B) 2 only
- (C) 4 only
- (D) 0 and 6
- (E) 2 and 4
- (F) 4 and 6
- (G) 2 and 6
- (H) 4 and 8

PTS

11. [5 pts] If $H(x) = \int_0^{2x} e^{3t^2} dt$, find $H'(1)$.

- (A) e^3
- (B) e^6
- (C) e^{12}
- (D) $2e^6$
- (E) $2e^{12}$

PTS

12. [5 pts] Let $f(x)$ be a continuous function defined for all real numbers. We are given that

$$\int_{-1}^1 f(x) dx = 3, \text{ and } \int_{-1}^2 f(x) dx = 5.$$

Calculate $\int_1^2 (3 - 2f(x)) dx$.

- (A) -1
- (B) 0
- (C) 1
- (D) 5
- (E) 7

PTS

13. [5 pts] Compute $\int_{-2}^0 1 + \sqrt{4 - x^2} \, dx$ by recognizing it as the area of a region made up of simple geometric shapes.

- (A) $\pi + 2$
- (B) $-\pi - 2$
- (C) 2π
- (D) $\pi + 4$
- (E) $-\pi$

_____ PTS

14. [5 pts] Compute the area bounded by $y = e^x$, $x = 0$, and $y = 2$.

- (A) $2 \ln(2) - 1$
- (B) $2 \ln(2) + 1$
- (C) $\ln(2) - 1$
- (D) $2 - \ln(2)$
- (E) $2 \ln(2)$

_____ PTS

15. [5 pts] A scientist is studying how the concentration of a chemical in a solution changes with temperature. The concentration C , measured in moles per liter, is modeled by the function

$$C(T) = \ln(T^2 + 5)$$

where T is the temperature in degrees Celsius. To make quick predictions without using a calculator, the scientist wants to use a linear approximation of $C(T)$ near $T = 2^\circ\text{C}$. Estimate the concentration at $T = 2.1^\circ\text{C}$.

- (A) $C(2.1) \approx \ln(9) + \frac{1}{45}$
- (B) $C(2.1) \approx \ln(9) + \frac{1}{90}$
- (C) $C(2.1) \approx \ln(9) + \frac{2}{45}$
- (D) $C(2.1) \approx \ln(9) + \frac{4}{81}$
- (E) $C(2.1) \approx \ln(9) + \frac{1}{9}$

PTS

16. [5 pts] $\int_0^2 e^{-x^2} dx$ is to be approximated using a Riemann sum with 8 subintervals of equal width, where the height of each rectangle is determined by the right endpoint of the subinterval. If i is an integer between 1 and 8, what is the area of the i^{th} rectangle?

- (A) $\frac{1}{8}e^{-\frac{i}{4}}$
- (B) $\frac{1}{8}e^{-\frac{i^2}{4}}$
- (C) $\frac{1}{4}e^{-\frac{i^2}{4}}$
- (D) $\frac{1}{4}e^{-\left(\frac{i}{4}\right)^2}$
- (E) $e^{-\frac{i}{4}}$

PTS

17. [5 pts] Evaluate $\int_0^1 \frac{x}{1+x^2} dx$.

- (A) $\ln(2)$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{4} \ln(2)$
- (D) $\arctan(1) = \frac{\pi}{4}$
- (E) $\frac{1}{2} \ln(2)$

_____ PTS

18. [5 pts] If $f(x)$ is continuous and it is known that $\int_0^2 f(x) dx = 6$, evaluate $\int_0^{\frac{\pi}{2}} f(2 \sin t) \cos t dt$.

- (A) 0
- (B) $\frac{3}{2}$
- (C) 2
- (D) 3
- (E) 4

_____ PTS

Part II: Free Response

Present all work leading to your final answer clearly and in a structured manner. Show all relevant steps and justify them.

19. [16 pts] Compute the following limits:

(a) $\lim_{t \rightarrow 1^-} \sqrt{2t + 5}$

_____ 4 PTS

(b) $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 4}$

_____ 4 PTS

(c) $\lim_{x \rightarrow 1} \frac{3^x - 3}{x - 1}$

_____ 4 PTS

(d) $\lim_{y \rightarrow +\infty} \sqrt{y^2 + 6y - 1} - y$

_____ 4 PTS

20. [10 pts]

- (a) Write the limit definition of the derivative of a function $f(x)$.

_____ 2 PTS

- (b) Use that definition to compute the derivative of $f(x) = \frac{1}{x+5}$. (No credit will be awarded for any other method.)

_____ 8 PTS

21. [24 pts] Consider the function

$$f(x) = \frac{x}{x^2 + 1}$$

with first derivative

$$f'(x) = \frac{1 - x^2}{(x^2 + 1)^2}$$

and second derivative:

$$f''(x) = \frac{2x(x^2 - 3)}{(x^2 + 1)^3}.$$

(a) Find the domain of $f(x)$.

_____ 2 PTS

(b) Find the x - and y - intercepts of $f(x)$.

_____ 2 PTS

(c) Does $f(x)$ possess any symmetry? (Even/Odd/Neither)

_____ 2 PTS

(d) Does $f(x)$ have any vertical or horizontal asymptotes? Justify.

_____ 4 PTS

(e) Using the first and/or the second derivative of $f(x)$, find all critical points.

_____ 2 PTS

(f) Using the first and/or the second derivative of $f(x)$, find all inflection points.

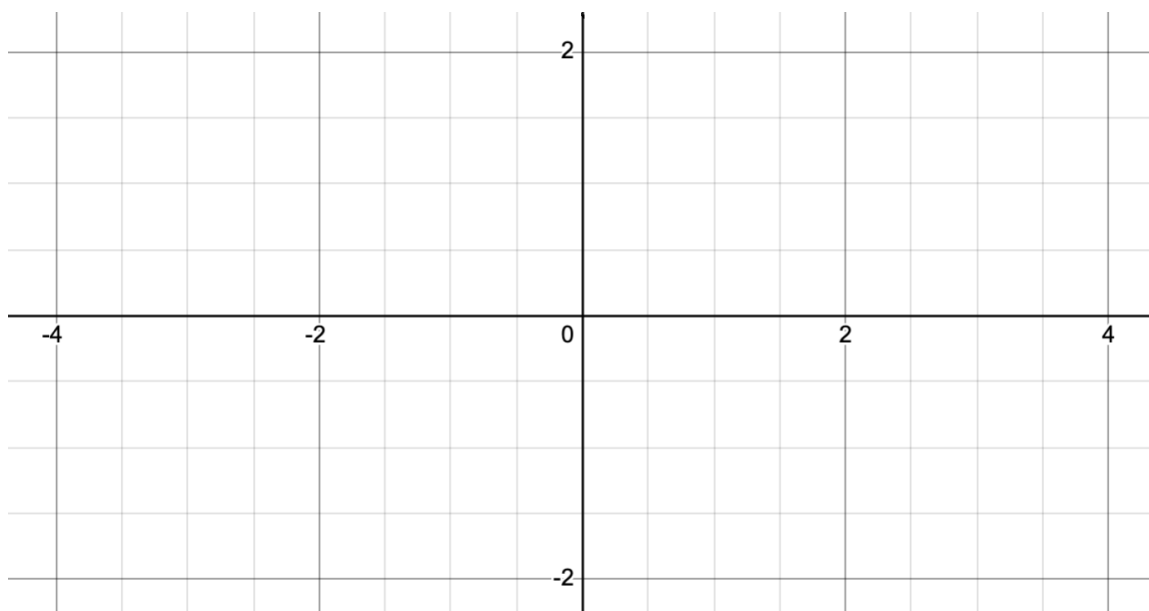
_____ 2 PTS

SUBTOTAL _____ 8 PTS

- (g) Organize the above findings in a table that encodes the signs of the first and second derivative of $f(x)$. Clearly state the interval(s) where the function is increasing, decreasing, concave up or concave down. If not applicable, state “none”.

_____ 6 PTS

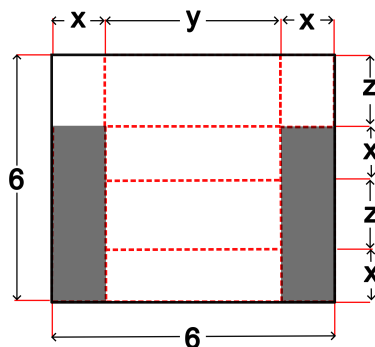
- (h) Sketch the function, putting in evidence intercepts, local extrema, inflection points and asymptotes, if any. Do not forget to label your graph.



_____ 4 PTS

SUBTOTAL _____ 10 PTS

22. [10 pts] A closed box with a lid is to be formed by trimming identical rectangles (shaded below) from two adjacent corners of a flat, 6-foot by 6-foot cardboard sheet with negligible thickness. The sides are then folded up to form the box. A diagram is provided here:



- (a) Write a formula for the volume of the box in terms of x along with a reasonable domain. Simplify the volume formula as much as you can.

4 PTS

- (b) Use calculus to find the value of x that gives the largest volume and justify your finding.

6 PTS



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