MATH 3100: Introduction to Mathematical Analysis

MATH 3100 is (since 2022) a required course in all branches of the math major. While some of its students proceed to MATH 4100, over 75% do not, so this course is the primary way that most of our majors will learn about the foundations of calculus.

Students in 3100 will (except in rare cases where an exemption has been granted) have already taken MATH 3200, but most of them will have little or no experience using the proof techniques that they learned in 3200 outside of that course. As such, students should be introduced gently to the necessary rigor of the course.

Broad topics of the course include:

- The real number system and how it differs from the rationals. Students should be able to reason about infs, sups, and the completeness axiom. They should also develop proficiency working with absolute values needed later for convergence arguments.
- Sequences and the ε-N definition of convergence. Students should be able to prove that specific sequences converge by explicit arguments that construct a suitable N for any given ε, and they should be able to write proofs of simple general results such as the statement that the sum of two convergent sequences is convergent.
- Functional limits and continuity. Students should know and be able to use the ε-δ definition of a limit and both the sequential and the ε-δ formulation of the definition of continuity.
- Key properties of continuous functions on closed intervals: the maximum value and intermediate value theorems. Students should understand both the statements of these and why they are nontrivial, and get some idea of what goes into their proofs. This includes the Bolzano-Weierstrass Theorem, and students should be able to reason about subsequences well enough to understand this statement.
- Differentiation. Students should know how the derivative is rigorously defined, know the statement of the mean value theorem, and be able to apply it to prove such results as the fact that a function with everywhere-zero derivative is constant. Generalizing the mean value theorem, the error estimate for Taylor polynomials should be covered, as preparation for MATH 4500.

One possible textbook is *Elementary Analysis: The Theory of Calculus* by Kenneth Ross (from the Springer UTM series; a pdf can be freely downloaded from within the UGA network). These notes by Malcolm Adams and Pete Clark from earlier versions of the course can be useful as a supplementary text. Here is a potential outline following Ross; sections not listed below could be added depending on time constraints. The suggestions for the number of days for each topic below are based on a MWF schedule. The total of the third column is 38, so this leaves seven days for exams, review, and slippage.

Ross	Topic	Days
Section 1	The natural numbers, including induction	1

Section 2	The rational numbers, and the irrationality of $\sqrt{2}$	1
Section 3	R as an ordered field; working with inequalities and absolute values	2
Sections 4-5	sup and inf, meaning of $\pm\infty$	2
Section 4	The completeness axiom, the Archimedean property, and denseness of Q	2
Section 7	Introduction to sequences, including ones defined recursively	1
Sections 7-8	Limits of sequences, and methods of proof for concrete examples	2
Section 9	Limit theorems for sequences	4
Section 10	Monotone and Cauchy sequences	2
Section 11	Subsequences and the Bolzano-Weierstrass theorem	1
Section 14	Definition and examples of series and their convergence	2
Section 14	Geometric series	1
Section 14	nth term test, harmonic series, comparison test	1
Section 14	Limit laws for sums, Cauchy criterion, absolute convergence	1
Section 14	Ratio and root tests	2
Section 15	p-series[1] and alternating series	2
Section 23	Basics of power series	1
Section 17	Continuity: definition and basic results	1
Section 18	Maximum and Intermediate Value Theorems and consequences	2
Section 20	Functional (ε-δ) limits	1
Section 28	Linear approximation and the definition of the derivative	1
Section 29	Rolle's Theorem and the Mean Value Theorem	1
Sections 28-29	Properties of the derivative, including the chain rule and the differentiability of inverse functions	2
Section 31	Taylor polynomials and Taylor's theorem with remainder.	2

[1] Note that Ross determines the convergence or divergence of p-series using the integral test, to which one could object on the grounds that integrals are not developed rigorously in the course. An alternative approach uses the Cauchy condensation test, discussed in Section 2.2.3 of the Adams-Clark notes.