Final Exam Review – Integrals

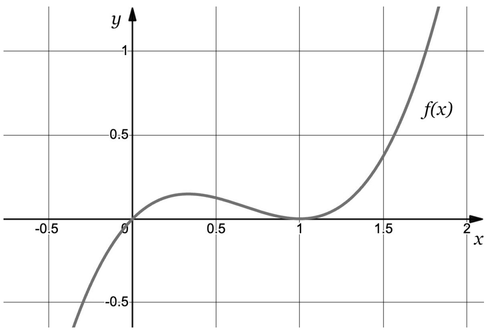
Openstax Sections: 4.10, 5.1-5.7, 6.1.

Exercises

- Evaluate the integral:

1. $\displaystyle\int_1^4 \frac{3t^2-\sqrt{t}}{t}\,dt$
2. $\displaystyle\int_{-1}^1 (3-6x^5)\,dx$
3. $\displaystyle\int_0^1 (1-x)^5\,dx$
4. $\displaystyle\int_0^1 \sin(2\pi x)\,dx$
5. $\displaystyle\int \frac{x^3}{x^4+1}\,dx$
6. $\displaystyle\int \sin x\cos(\cos x)\,dx$
7. $\displaystyle\int e^x\sqrt{1+e^x}\,dx$
8. $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$

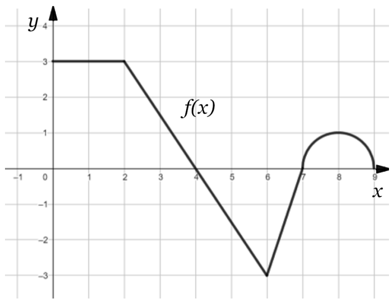
- If f is continuous and $\displaystyle\int_0^4 f(x)\,dx = 6$, find $\displaystyle\int_0^2 f(2x)\,dx$.
- Find $f(\theta)$ if $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 1$, and $f'(0) = 2$.
- Without computing, determine which of the integrals $\displaystyle\int_1^4 \sqrt{x}\,dx$ or $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}\,dx$ has the larger value. Justify your answer.
- Given f, sketch the graph of its antiderivative F that passes through the point $\left(\frac{1}{2},\frac{1}{2}\right)$.



- Evaluate:

1. $\displaystyle\frac{d}{dx}\int_0^1 e^{x^3}\,dx$
2. $\displaystyle\frac{d}{dx}\int_e^x e^{t^3}\,dt$
3. $\displaystyle\int_0^1 \frac{d}{dx}e^{x^3}\,dx$
4. $\displaystyle\frac{d}{dx}\int_\pi^{x^2} e^{t^3}\,dt$

- A miniature Christmas train moves back and forth along a straight track in a holiday display. Its velocity is $v(t)$ (in feet per second) and its acceleration is $a(t)$ (in feet per second squared), measured at time t seconds.
  1. What does the integral $\displaystyle\int_{10}^{30} v(t)\,dt$ represent in the context of the train's motion?
  2. What does the integral $\displaystyle\int_{10}^{30} |v(t)|\,dt$ represent?
  3. What does the integral $\displaystyle\int_{10}^{30} a(t)\,dt$ represent?
- Consider $\displaystyle\int_{0}^{3} (2x+3)\,dx$.
  1. Estimate the above integral using Riemann sum with three subintervals, taking the sample points to be left endpoints.
  2. Write the above integral as a limit of Riemann sums, taking the sample points to be right endpoints and evaluate the sum.
  3. Check your answer using the Fundamental Theorem of Calculus.
- Determine the value of the integral $\int_1^2 (4f(x)-2x)\,dx$ given that $\int_{-1}^{0} f(x)\,dx = 5$, $\int_{-1}^{1} f(x)\,dx = 10$, and $\int_0^2 f(x)\,dx = 24$.
- Use the graph of a function f below to answer the following questions:



  1. Evaluate $\displaystyle\int_0^9 f(x)\,dx$.
  2. Find the area (not net area) bounded by the graph of $f(x)$ and the x-axis on $[0,9]$.
  3. If $F(t) = \displaystyle\int_0^t f(x)\,dx$, find $F'(1)$.
- Find the area of the region bounded by:
  1. $y = \frac{1}{x}, \quad y = x^2, \quad y = 0, \quad x = e.$
  2. $x + y = 0, \quad x = y^2 + 3y.$