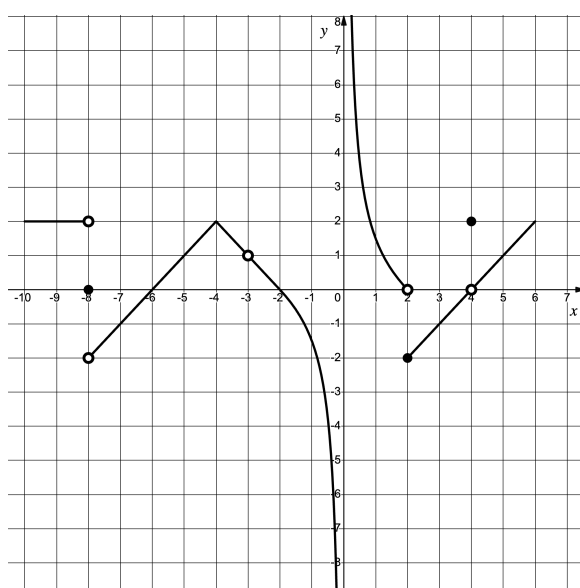


Final Exam Review – Limits

Openstax Sections: 2.1-2.4, 3.1, 3.2, 4.6, 5.2.

EXERCISES

- ♦ Consider the graph of $f(x)$ below. Compute the following quantities when possible. When impossible, explain why not.



1. $\lim_{x \rightarrow 4} f(x)$
 2. $\lim_{x \rightarrow 2^+} f(x)$
 3. $\lim_{x \rightarrow -3^-} \frac{f(x) - 1}{x + 3}$
 4. $\lim_{x \rightarrow 5} \frac{f(x) - f(5)}{x - 5}$
 5. $\lim_{x \rightarrow -8} f(x)$
 6. $\lim_{x \rightarrow 0^+} f(x)$
 7. Average rate of change of $f(x)$ on $[-4, 4]$.
 8. Instantaneous rate of change of $f(x)$ at $x = -5$.
 9. Instantaneous rate of change of $f(x)$ at $x = 4$.
 10. For what values of x is $f(x)$ discontinuous? Classify each discontinuity as removable, jump or infinite using limits.
- ♦ Evaluate the following limits:

1. $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 2}$
2. $\lim_{x \rightarrow 2^+} \sqrt{x + 7} - 2x$
3. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$
4. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 - 8}$
5. $\lim_{x \rightarrow \pi^-} \ln(\sin x)$
6. $\lim_{x \rightarrow \infty} \frac{x^2 - 1}{x - 2}$
7. $\lim_{x \rightarrow -\infty} \frac{x^2 - 1}{x^5 - 2x^2 + 1}$
8. $\lim_{x \rightarrow \infty} \sqrt{x^2 + 4x + 1} - x$
9. $\lim_{x \rightarrow 1} \frac{1}{x - 1} - \frac{1}{x^2 + 4x - 5}$
10. $\lim_{u \rightarrow 1} \frac{\sin(u - 1)}{2u - 2}$
11. $\lim_{x \rightarrow -\infty} \frac{e^x + 1}{1 - x}$
12. $\lim_{x \rightarrow \infty} \frac{x + 1}{e^x + 1}$
13. $\lim_{x \rightarrow 0^+} x^2 \ln x$
14. $\lim_{x \rightarrow \infty} \sqrt{x}^{\frac{1}{x}}$

♦ Let

$$f(x) = \begin{cases} ax^2 - 3x & \text{if } x \leq 1, \\ bx + 4 & \text{if } x > 1, \end{cases}$$

For what value(s) of a and b , if any, is $f(x)$ differentiable everywhere?

♦ Find all values of a such that f is continuous on $\mathbb{R}$:

$$f(x) = \begin{cases} x + 1 & \text{if } x \leq a, \\ x^2 & \text{if } x > a. \end{cases}$$

♦ What type of discontinuity do the following functions have? Justify your response using limits.

$$1. y = \frac{2}{1 - 3x}$$

$$2. y = \frac{x^2 - 4}{x - 2}$$

♦ Does $y = \frac{\sqrt{x^2 + 4}}{1 + 3x}$ have any vertical or horizontal asymptotes? Justify your response using limits.

♦ Recognize this limit to compute it: $\lim_{z \rightarrow 0} \frac{e^{\sin(\frac{\pi}{4} + z)} - e^{\frac{\sqrt{2}}{2}}}{z}$.

♦ Write the integral $\int_1^3 (2x - 1) dx$ as the limit of a Riemann sum, taking the sample points to be the right endpoints of the subintervals, and using $a = 1$ and $b = 3$.

♦ Compute $\lim_{h \rightarrow 0} \frac{1}{h} \int_1^{1+h} \sqrt{1 + e^{t^2}} dt$.