CALCULUS I REVIEW

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4. Applications

(1) Find the local and absolute extrema of $y = \dfrac{3x-4}{1+x^2}$ over $[-2, 2]$.

(2) Consider the function $f(x) = x^2e^{-x}$ on $[-1, 3]$.
   (a) Find **absolute maximum value** and the **absolute minimum value** of $f(x)$. *Leave the answer in terms of e.*
   (b) At **what values of x** does the function attain a local maximum and a local minimum? *Leave the answer in terms of e.*

(3) Suppose g is a differentiable function satisfying $g(1) = 1$ and $g'(1) = 3$. Let $f(x) = \dfrac{g(x)}{x-2}$. Determine $f'(1)$.

(4) Consider the function $y = \dfrac{x}{1-x^2}$. Study it (i.e. state its domain, find $x-$ and $y-$ intercepts, see if it possesses any symmetry, find vertical, horizontal and slant asymptotes, compute first and second derivatives and find critical as well as potential inflection points, organize the latest findings in a table and place in evidence the intervals of increase and decrease, local extrema, intervals of concavity as well as inflection points, then use all this information to sketch the graph).

(5) Idem for $y = x\sqrt{x+2}$ and $f(x) = \frac{x^2}{x^2+3}$.

(6) A tank of water in the shape of a cone is being filled with water at a rate of 4 m^3/sec. The base radius of the tank is 12 m and the height of the tank is 6 m. At what rate is the depth of the water in the tank changing when the radius of the top of the water is 6 m?

(7) A 50 ft ladder is placed against a large building. The base of the ladder slips at the rate of 3 ft per minute. Find the rate of change of the height of the top of the ladder above the ground at the instant when the base of the ladder is 30 ft from the base of the building.

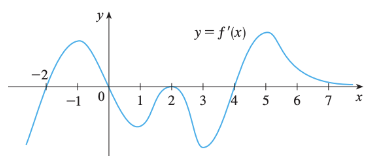
(8) The altitude of a triangle is increasing at a rate of $1\,\text{cm/min}$, while the area of the triangle is increasing at a rate of $2\,\text{cm}^2/\text{min}$. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is $100\,\text{cm}^2$.

(9) If $1200\,\text{cm}^2$ of material is available to make a box with a square base and an open top, find the

largest possible volume of the box.

(10) We plan to enclose a rectangular field with a fence. The costs of the sides vary: the vertical sides are \$4/ft, the bottom costs \$2/ft, and the top costs \$6/ft. Given a budget of \$800 available, find the dimensions of the field that will maximize the area enclosed by the fence.

(11) The graph of the **derivative** of a function $f(x)$ is given below.



(a) For which intervals is the function growing or declining?
(b) At which values of x does f attain a local maximum or minimum?
(c) Sketch a possible graph of f.

(12) For what values of the constants a and b is $(1, 3)$ a point of inflection of the curve $y = ax^3 + bx^2$.

(13) Find a point on the hyperbola $xy = 8$ that is closest to the point $(3, 0)$.

(14) Use linear approximation to $f(x) = xe^{2x-10}$ at $x = 5$ to estimate by hand $f(5.1)$.