By providing my signature below, I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (print): ______________________ Name (sign): ______________________

Rubric

Student ID (81#): ______________________

Instructor's Name: ______________________ Class Time: ______________________

Problem Number	Points Possible	Points Earned
1	20	
2	10	
3	12	
4	40	
5	30	
6	20	
7	18	
8	12	
9	28	
Total:	190	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion, then state the theorem used by name.
- Please provide **neat**, organized work to ensure partial credit.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Cell phones and smart watches are NOT allowed; smart devices (including smart watches and cell phones) may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.
- You are only allowed to use a **TI-30XS Multiview** calculator; the name must match exactly. No other calculators are permitted, and sharing of calculators is not permitted.
- You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem.

Formula Sheet

- Circles
  - area: $A = \pi r^2$
  - circumference: $C = 2\pi r$
  - Equation of the circle of radius r centered at (h, k): $(x-h)^2 + (y-k)^2 = r^2$
- Rectangles
  - area: $A = lw$
  - perimeter: $P = 2l + 2w$
- Cylinder
  - volume: $V = \pi r^2 h$
  - surface area: $S = 2\pi r^2 + 2\pi r h$ (includes base and lid)
- Rectangular prisms
  - volume: $V = lwh$
  - surface area: $S = 2lw + 2wh + 2lh$ (includes top and base)
- Circular cone
  - volume: $V = \frac{1}{3}\pi r^2 h$
- Sphere
  - volume: $V = \frac{4}{3}\pi r^3$

1. Determine the following limits; show work or briefly explain your thinking on each one. If you apply L'Hopital's Rule, indicate where you have applied it and why you can apply it. If your final answer is "does not exist," ∞, or, $-\infty$, briefly explain your answer. (You will not receive full credit for a "does not exist" answer if the answer is ∞ or $-\infty$.)

_______ (a) [4 pts] $\lim_{x\to 2} \sqrt[3]{x^2 - 6x - 7}$

$= \sqrt[3]{2^2 - 6(2) - 7}$

$= \sqrt[3]{4 - 12 - 7}$

[redacted]

[redacted]

[redacted]

(b) [6 pts] $\lim_{x\to 0} \dfrac{\sin(x) - x}{1 - \cos(x)} \longrightarrow \dfrac{0}{0}$

____

[redacted] $\overset{LH}{=} \lim_{x\to 0} \dfrac{\cos(x) - 1}{\sin(x)} \longrightarrow \dfrac{0}{0}$

[redacted] $\overset{LH}{=} \lim_{x\to 0} \dfrac{-\sin(x)}{\cos(x)} \rightleftarrows \dfrac{0}{1} = 0$

[redacted]

[redacted]

(c) [4 pts] $\lim_{x\to\infty} e^{-4x} + \ln(x)$ $= 0 + \infty = \infty$

$\uparrow$

[redacted]

[redacted]

(d) [6 pts] $\lim_{x\to 0^+} \left(\frac{1}{x} + \frac{5}{x(x-5)}\right)$ $\longrightarrow \infty - \infty$

$= \lim_{x\to 0^+} \frac{x-5+5}{x(x-5)}$ [redacted]

[redacted]

$= \lim_{x\to 0^+} \frac{x}{x(x-5)}$ [redacted]

$= \lim_{x\to 0^+} \frac{1}{x-5}$ [redacted]

$= \frac{-1}{5}$ [redacted]

[redacted] [redacted] [redacted]

2. [10 pts] Use the **limit definition of the derivative** to determine the derivative of

$$f(x) = \sqrt{x+1}.$$

Your answer should be $f'(x) = \dfrac{1}{2\sqrt{x+1}}$. No points will be awarded for the application of differentiation rules (and L'Hopital's Rule is not allowed.)
Show all steps.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

[redacted]

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\left(\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}\right)$$

[redacted]

$$= \lim_{h\to 0} \frac{x+h+1-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})}$$

[redacted]

$$= \lim_{h\to 0} \frac{x+h+1-x-1}{h(\sqrt{x+h+1}+\sqrt{x+1})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})}$$

[redacted]

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

[redacted]

$$= \frac{1}{2\sqrt{x+1}}$$

[redacted]

$$= \lim_{h\to 0} \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$$ 2pts

scratch work:

2pts $\dfrac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\left(\dfrac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}\right)$

2pts $= \dfrac{x+h+1-x-1}{h(\sqrt{x+h+1}+\sqrt{x+1})}$

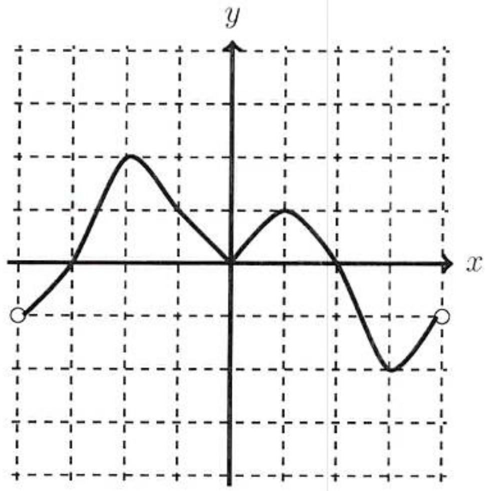
$= \dfrac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})}$

2pts $= \dfrac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$ 2pts

$$= \frac{1}{2\sqrt{x+1}}$$

3. The graph below is the **graph of the derivative of f** (and not the graph of $f(x)$). Use it to answer the questions that follow, keeping in mind that the graph below is the graph of the **DERIVATIVE** of f, $y = f'(x)$. The domain of the function $f(x)$ is the interval $(-4, 4)$. The grid lines are one unit apart.



(a) [2 pts] Determine whether $f(x)$ (not the derivative) is increasing, decreasing, or neither on the interval $(-2, 0)$.

$f'(x) > 0 \Rightarrow f$ is increasing [redacted]

[redacted]

(b) [5 pts] $(3, 4)$ is a point on the graph of $f(x)$. Write an equation of the line tangent to $f(x)$ at the point $(3, 4)$.

slope: -2 [redacted]

$y - 4 = -2(x - 3)$ or $y = -2x + 10$

[redacted]

(c) [5 pts] Determine the x-coordinate(s) of all relative (local) maxima of $f(x)$ on the interval $(-4, 4)$. Use $f'(x)$ to justify your answer.

f': $\ominus$ -3 $\oplus$ 0 $\oplus$ 2 $\ominus$

$x=-3$, $x=0$, $x=2$

relative max @ $x=2$

[redacted]

[redacted] for [redacted]

4. Determine $\frac{dy}{dx}$ for each equation below. Remember to use correct notation to write your final answer. You do not need to simplify your final answer.

(a) [5 pts] $y = \frac{2}{x^5} - \sqrt[3]{x}$ $= 2x^{-5} - x^{1/3}$

$$\frac{dy}{dx} = \underbrace{-10x^{-4}}_{\text{[redacted]}} \underbrace{- \frac{1}{3}x^{-2/3}}_{\text{[redacted]}}$$

(b) [6 pts] $y = \frac{e^{-5x}}{\sin(x^2)}$

$$\frac{dy}{dx} = \frac{\sin(x^2)\overbrace{(-5e^{-5x})}^{\text{[redacted]}} - e^{-5x}\overbrace{\cos(x^2)(2x)}^{\text{[redacted]}}}{(\sin(x^2))^2}$$

[redacted]

(c) [6 pts] $y = \ln(x)\tan(x)$ [redacted]

$$\frac{dy}{[redacted]} = \ln(x)\overbrace{\sec^2(x)}^{\text{[redacted]ts}} + \tan(x)\left(\frac{1}{x}\right)$$

[redacted]

(d) [8 pts] $y = \sqrt{\cos^2(x) + 2x}$

2pts

$$\frac{dy}{dx} = \frac{1}{2}\underbrace{(\cos^2(x) + 2x)}_{\text{[redacted]ts}}^{-1/2}\left(\underbrace{2\cos(x)\sin(x)}_{\text{[redacted]}} + 2\right)$$

[redacted] [redacted]

Determine $\frac{dy}{dx}$ for each equation below. Remember to use correct notation to write your final answer. You do not need to simplify your final answer.

[redacted] : [redacted]

(e) [10 pts] $x^3 + y^3 = 2x + x\ln(y)$

$$3x^2 + 3y^2\frac{dy}{dx} = 2 + x\left(\frac{1}{y}\right)\frac{dy}{dx} + \ln(y)$$

2 [redacted]

2pts $$3y^2\frac{dy}{dx} - \frac{x}{y}\frac{dy}{dx} = 2 + \ln(y) - 3x^2$$

$$\frac{dy}{dx}\left(3y^2 - \frac{x}{y}\right) = 2 + \ln(y) - 3x^2$$

[redacted]

$$\frac{dy}{dx} = \frac{2 + \ln(y) - 3x^2}{3y^2 - \frac{x}{y}}$$

1 pt

-6 pts [redacted] no [redacted]

(f) [5 pts] $y = \int_2^x \left(\sqrt{e^t} + \frac{1}{1+4t^2}\right)dt$

$$\frac{dy}{dx} = \sqrt{e^x} + \frac{1}{1+4x^2}$$

[redacted]

or

$$\frac{d}{dx}\int_2^x \left(e^{t/2} + \frac{1}{1+4t^2}\right)dt$$

$$= \frac{d}{dx}\left[2e^{t/2} + \frac{\arctan(2t)}{2}\right]_2^x$$

$$= \frac{d}{dx}\left(2e^{x/2} + \frac{\arctan(2x)}{2}\right) - \left(2e^{2/2} + \frac{\arctan}{2}\right.$$

3p

$$= \frac{2e^{x/2}}{2} + \frac{2}{2}\left(\frac{1}{1+(2x)^2}\right) - 0$$

$$= e^{x/2} + \frac{1}{1+4x^2}$$

[redacted]

5. Evaluate the following. You do not need to simplify your answers.

______ (a) [8 pts] $\int \left(5 + \frac{1}{3x^{1/5}} - \frac{9}{x} + x^2\right) dx$ $= \int \left(5 + \frac{1}{3}x^{-1/5} - \frac{9}{x} + x^2\right) dx$

1pt

←1pt

$= 5x + \dfrac{\frac{1}{3}x^{4/5}}{\frac{4}{5}} - 9\ln|x| + \dfrac{x^3}{3} + C$

1[redacted] [redacted] [redacted] [redacted] [redacted] [redacted]

______ (b) [8 pts] $\int \left(\sin(x) - \frac{1}{\sqrt{1-x^2}}\right) dx$

[redacted] 2pts 1pt [redacted] 2pts

$= -\cos(x) - \arcsin(x) + C$

or

$= -\cos(x) + \arccos(x) + C$

[redacted]

Evaluate the following. You do not need to simplify your answers.

______ (c) [6 pts] $\int \frac{3\cos(\ln(x))}{x}dx$

$= \int 3\cos(u)\,du$ [redacted]

$= 3\sin(u) + C$ [redacted]

$= 3\sin(\ln(x)) + C$ [redacted]

1pt ~~2pts~~ $u = \ln(x)$

1pt $du = \frac{1}{x}dx$

[redacted]

-1 pt [redacted] [redacted] [redacted]

(d) [8 pts] $\int_{-1}^{0} e^{-3x+1}dx$

______

$u = -3x + 1$

$du = -3\,dx$ [redacted]

$-\frac{du}{3} = dx$

$u(-1) = -3(-1) + 1 = 4$ [redacted]

$u(0) = 1$

2 [redacted] s

$\int_4^1 e^u \frac{du}{-3} = -\frac{1}{3}\left[e^u\right]_4^1$ [redacted]

$= -\frac{1}{3}e + \frac{1}{3}e^4$ 1pt

[redacted] b [redacted]

[redacted] 1pt

$= \left[-\frac{1}{3}e^{-3x+1}\right]_{-1}^{0}$

$= -\frac{1}{3}e + \frac{1}{3}e^4$

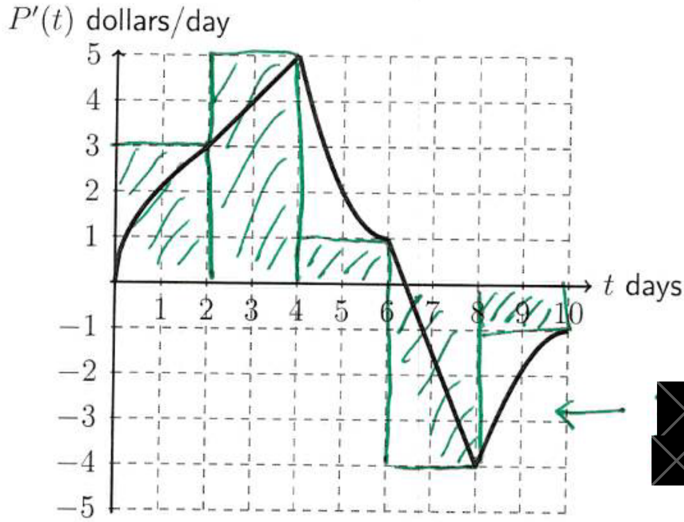
[redacted] [redacted] [redacted]

[redacted]

6. Suppose $P(t)$ is the price of a stock, in dollars, t days after the company goes public. Although we've lost the data for the actual stock prices, we did manage to find data for the **rate of stock price change** $P'(t)$ for the first 10 days since the stock opened!

That data is graphed below. Grid markings represent 1 unit.

Graph of $P'(t)$: the **DERIVATIVE** of $P(t)$



[redacted]

______ (a) [5 pts] On the graph above, draw 5 rectangles which represent a Riemann sum for the area between $P'(t)$ and the t-axis on $[0, 10]$, using right-endpoints.

$\Delta x = \frac{10-0}{5} = 2$ [redacted]

______ (b) [10 pts] Compute the Riemann sum that you drew in part (a). You may leave your answer unsimplified.

$A \approx 2(3) + 2(5) + 2(1) - 2(4) - 2(1) = 8$

[redacted] ts

______ (c) [5 pts] Using the Net Change Theorem, interpret the meaning of the integral $\int_0^{10} P'(t)\,dt$ **in the context of the problem.**

$\int_0^{10} P'(t) = \underbrace{P(10) - P(0)}_{2\text{pts}}$ is the change in stock price between day 0 and day 10.

[redacted]

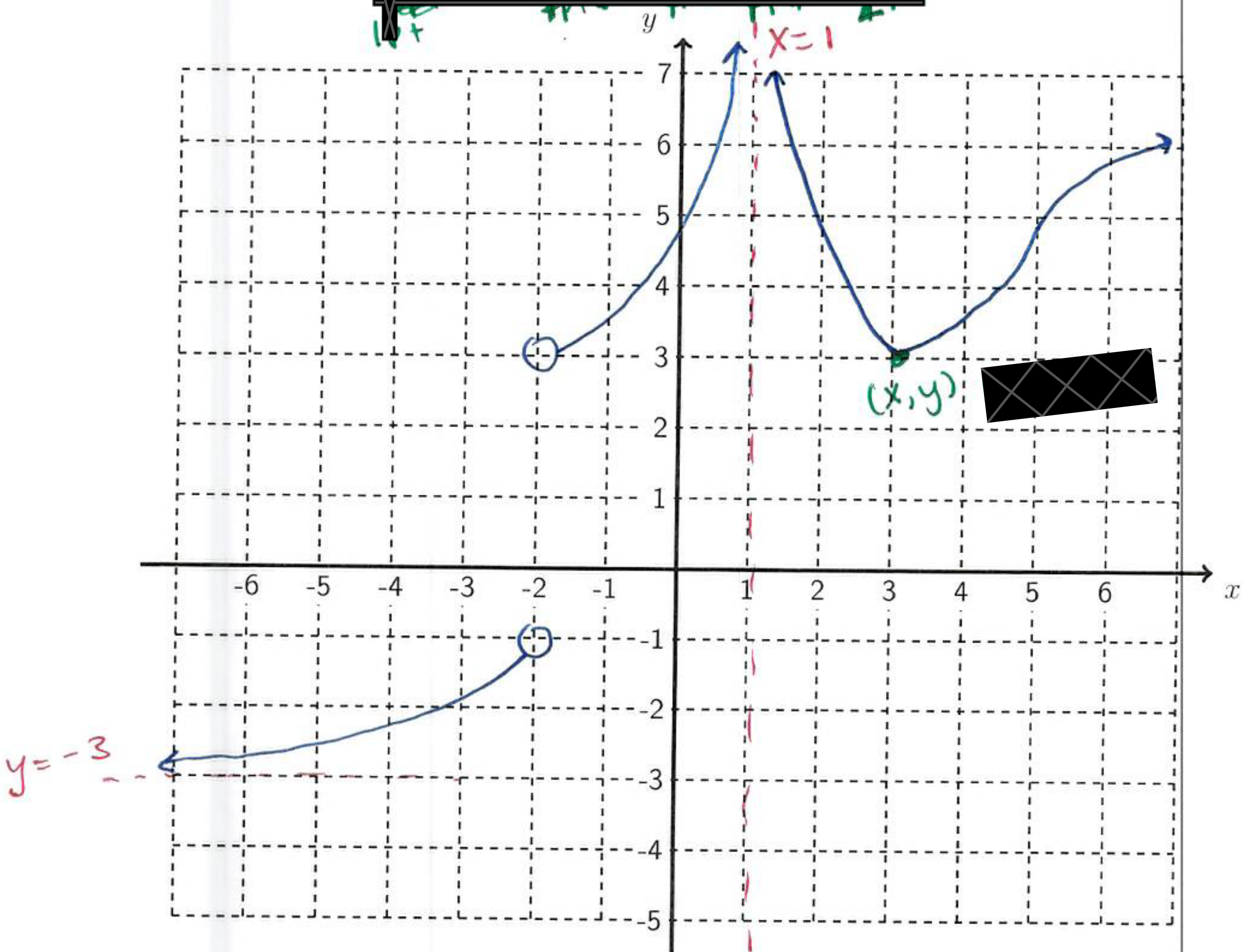
7. (a) [15 pts] Sketch the graph of a function $y = f(x)$ satisfying all of the following criteria:

- Its domain is $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$. [redacted]
- $\lim_{x \to -2^-} f(x) = -1$ and $\lim_{x \to -2^+} f(x) = 3$ [redacted]
- The line $x = 1$ is a vertical asymptote. [redacted]ts
- $\lim_{x \to \infty} f(x) = \infty$ 1pt
- $\lim_{x \to -\infty} f(x) = -3$ 2pts
- The sign chart of the first and second derivatives is below:

interval	$(-\infty, -2)$	$(-2, 1)$	$(1, 3)$	$(3, 5)$	$(5, \infty)$
sign f'	+	+	−	+	+
sign f''	+	+	+	+	−

[redacted]

[redacted] 1pt 1pts 1pts 1pt 2pts



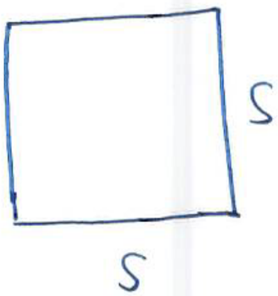
(b) [3 pts] On your sketch, label all asymptotes and points (x, y) where relative extrema exist.

[redacted]

[redacted] @ [redacted]

P[redacted])

8. [12 pts] Each side of a square is increasing at a rate of 6 cm/sec. At what rate is the area of the square increasing when the area of the square is 16 cm^2 ? Your final answer should be a complete sentence with appropriate units.



[redacted] picture

$$\frac{ds}{dt} = 6 \text{ cm/sec}$$

$$A = 16 \text{ cm}^2$$

$$s = \sqrt{16} = 4 \text{ cm}$$ [redacted]

Find? $\frac{dA}{dt}$

$$A = s^2$$ [redacted]

$$\frac{dA}{dt} = 2s \frac{ds}{dt}$$ [redacted]

$$\frac{dA}{dt} = 2(4)(6) = 48 \text{ cm}^2/\text{s}$$

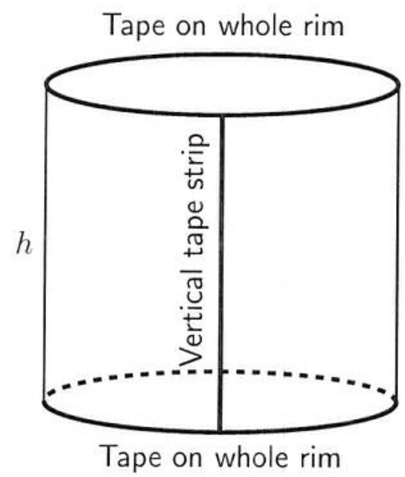
[redacted]

The area of the square is increasing at 48 cm^2/sec when the area is 16 cm^2.

↑

[redacted]

9. A cylindrical-shaped tube with radius r and height h is made of a long rolled-up sheet plus two circular lids, held closed with tape. We need to put tape around the rims of both lids, as well as one long strip of tape down the side of the tube. (See the labels on the figure on the below.)



_______ (a) [10 pts] Given that we have 20π inches of tape to seal this tube shut, write a function for the **VOLUME** of the tube **as a function of the radius r.**

$V = \pi r^2 h$ [redacted]

$V(r) = \pi r^2 (20\pi - 4\pi r)$ 2pts

$20\pi = 2(2\pi r) + h$ 4pts

$20\pi = 4\pi r + h$

$20\pi - 4\pi r = h$ [redacted]

_______ (b) [3 pts] Determine an appropriate domain for your function from (a). Briefly explain your reasoning.

$r \geq 0, \quad h \geq 0$

$20\pi - 4\pi r \geq 0$

$20\pi \geq 4\pi r$

$\frac{20\pi}{4\pi} \geq r$

$5 \geq r$

Domain: $[0, 5]$ [redacted]

or

$(0, 5)$

[redacted] pt

(c) [15 pts] **NOTE: Your work from parts (a) and (b) will not be used in this question.**

[redacted]

The amount of tape needed to create a 700π in^3 tube with radius r is given by

$$T(r) = 4\pi r + \frac{700}{r^2}.$$

The domain of $T(r)$ is $(0, \infty)$. Determine the radius r of the cylinder that will minimize the amount of tape needed. *Justify, using methods of calculus, why your dimensions yield the absolute minimum.*

$$T'(r) = 4\pi - \frac{1400}{r^3} = \frac{4\pi r^3 - 1400}{r^3}$$ [redacted]

$T'(r)$ dne: $r = 0$, not in domain

$T'(r) = 0$:

$4\pi r^3 - 1400 = 0$ [redacted]

$4\pi r^3 = 1400$

$r^3 = \frac{1400}{4\pi}$

$r = \sqrt[3]{\frac{1400}{4\pi}} \approx 4.8$

[redacted]

Optimization Method:
- First Deriv. Test or 2nd Deriv

T': $-$ | $+$ at $\sqrt[3]{\frac{1400}{4\pi}}$ [redacted]

T: ↘ ↗

$\Rightarrow$ abs. min @ $r = \sqrt[3]{\frac{1400}{4\pi}}$

just [redacted]

The radius must be $\sqrt[3]{\frac{1400}{4\pi}}$ in to minimize the amount of tape needed.

[redacted]

[redacted]

This page is extra space for work. **Do not detach this page.** If you want us to consider the work on this page, you should print your name, instructor and class meeting time below. For the problems where you want us to look at this work, please write "see last page" next to your work on the problem page so that we know to look here.

Name (print): ______________________ Instructor (print): ______________________

Class Meeting Time: ______________