Division of Academic Enhancement
UNIVERSITY OF GEORGIA
dae.uga.edu/tutoring/math-statistics

Department of Mathematics
Franklin College of Arts and Sciences
UNIVERSITY OF GEORGIA
math.uga.edu/studyhall

MATH 1113 Exam 2 Review

Fall 2017

Topics Covered

Section 3.1: Inverse Functions

Section 3.2: Exponential Functions

Section 3.3: Logarithmic Functions

Section 3.4: Properties of Logarithms

Section 3.5: Exponential and Logarithmic Equations and Applications

Section 3.6: Modeling with Exponential and Logarithmic Functions

Section 3.1: Inverse Functions

Functions
Recall: A function from A to B is a rule of correspondence that assigns each element of A to exactly ONE element of B.

A one to one function from A to B is a rule of correspondence that assigns each element of A to exactly ONE element of B ***AND*** each element of B is assigned to exactly ONE element of A. All one to one functions are invertible.

Vertical line test
A graph in the xy-plane represents a function $y = f(x)$ provided that any vertical line intersects the graph in at most one point.

Horizontal line test
A graph of a *function* in the xy plane represents a one to one function $y = f(x)$ provided that any horizontal line intersects the graph in at most one point.

Inverse Function
A function $y = f^{-1}(x)$ is an inverse function of $f(x)$ if the following properties hold true.

- The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected over the line $y = x$.
- The compositions of $f(x)$ and $f^{-1}(x)$ returns x. $f^{-1}\big(f(x)\big) = x = f\big(f^{-1}(x)\big)$
- The x and y coordinates of any point on $f(x)$ are reversed as points on $f^{-1}(x)$. For example, if (4,3) is a point on $f(x)$, then (3,4) is a point on $f^{-1}(x)$.
- The domain of $f(x)$ is the range of $f^{-1}(x)$ and the range of $f(x)$ is the domain of $f^{-1}(x)$
- All one to one functions have inverses.
- Functions that are not one to one functions *may* have inverses if the domain of $f(x)$ can be restricted to make it one to one. (Example $y = x^2$)

Finding the inverse function algebraically

- Write the original equation of $f(x)$ as $y = f(x)$
- Rewrite the equation replacing y with x and vice versa
- Solve for y
- If the new equation is a function, then it is the inverse of $f(x)$ (Note a domain restriction may be required)

Examples

1. Let $f(x)$ be an invertible function. Complete the following table

x	-2	-1	0	1	4	5	6
$f(x)$	10	7	4	0	-3	-4	-7

x							
$f^{-1}(x)$							

2. Use the function $f(x)$ to answer the following:

$$f(x) = \frac{4}{x-2}$$

(a) Find the inverse of $f(x)$. (Find $f^{-1}(x)$)

(b) Determine the domain and range of $f(x)$.

(c) Is $f(x)$ a one to one function? Why do you think so?

3. Answer the following about inverses.

(a) Is $f(x) = x^2$ invertible? If so, where and find its inverse.

(b) What about $f(x) = (x-2)^2 + 5$?

(c) What can we say about the invertibility of quadratic functions?

Section 3.2: Exponential Functions

Properties of Exponents

$b^0 = 1$ $\qquad$ $b^n b^m = b^{n+m}$ $\qquad$ $\dfrac{b^n}{b^m} = b^{n-m}$ $\qquad$ $(b^n)^m = b^{nm} = (b^m)^n$

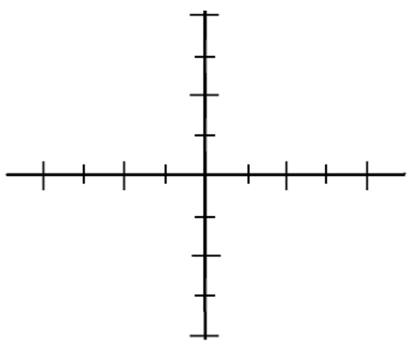
$(ab)^n = a^n b^n$ $\qquad$ $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ $\qquad$ $b^{-n} = \dfrac{1}{b^n}$ $\qquad$ $\dfrac{1}{b^{-n}} = b^n$

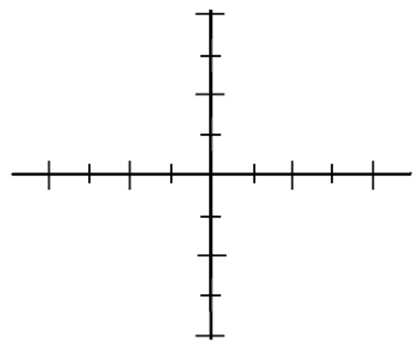
Exponential Functions vs. Power Functions
A power function is a variable raised to a constant power like $y = x^2$ (variable is the base)

An exponential function is a constant base raised to a variable power like $y = 2^x$
(variable is in the exponent)

The general form of an exponential function is $y = b^x$ where b is the base of the function and $b \neq 1$.

- If $0 < b < 1$, then $y = b^x$ will decrease exponentially
- If $1 < b$, then $y = b^x$ will increase exponentially





Examples

4. Solve for x.

$$27^{3x+2} = 81^{4x+5}$$

5. Solve for x.

$$2^x = 5^{4x+3}$$

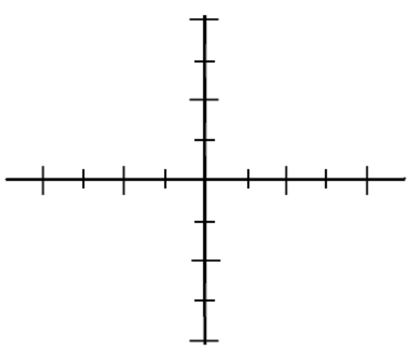
Section 3.3: Logarithmic Functions

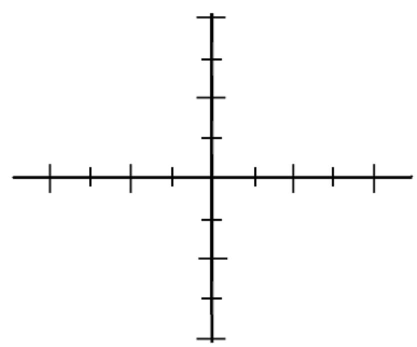
Logarithmic Functions

A logarithmic function is the inverse function of an exponential function.

$$y = b^x \;\rightarrow\; \log_b y = x$$

$y = b^x$	$\log_b y = x$
$D: (-\infty, \infty)$	$D: (0, \infty)$
$R: (0, \infty)$	$R: (-\infty, \infty)$





Examples

6. Find all the solutions to the equation $e^{4x-1} = 6$.

7. Find all the solutions to the equation $e^{2x} - 10e^{x} + 21 = 0$.

8. Find all the solutions of the equation $ln(x^2) = 5$.

9. Determine the domain of the function $f(x) = \log(x^2 - 9)$

Section 3.4: Properties of Logarithms

Properties of Log Functions

$$\log_b 1 = 0 \qquad \log_b b = 1 \qquad b^{\log_b x} = x$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n \qquad \log_b(mn) = \log_b m + \log_b n \qquad \log_b(m^r) = r\log_b m$$

Remember, $\ln x = \log_e x$ and $\log x = \log_{10} x$. So, the same properties hold for $\ln x$ and $\log x$.

Change of Base Formula

$$\log_{\mathrm{b}} x = \frac{\log_a x}{\log_a b} \ for\ all\ a > 0$$

Examples

10. Solve for x.

$$\log(x + 21) + \log(x) = 2$$

11. Solve for x.

$$\ln(3x + 7) = 3 + \ln(x)$$

12. Determine the domain of the function $f(x) = \sqrt{1 - e^{3x-9}}$

13. Find all the solutions of the equation $\log x + \log(x + 1) = \log 6$

<u>Section 3.5: Exponential and Logarithmic Equations and Applications</u>

<u>Compound Interest</u>

Compound Interest Formula (Interest Compounded n times per Year)

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A = Amount\ of\ money\ at\ time\ t$
$P = intial\ amount\ invested\ (principal)$
$r = annual\ interest\ rate$
$t = time\ in\ years$
$n = number\ of\ times\ per\ year$

Compound Interest Formula (Interest Compounded Continuously)

$$A = Pe^{rt}$$

$A = Amount\ of\ money\ at\ time\ t$
$P = intial\ amount\ invested\ (principal)$
$r = annual\ interest\ rate$
$t = time\ in\ years$

Examples

14. Scrooge McDuck wants to expand his wealth even further. The First Bank of Duckburg will give him an annual interest rate of 2% compounded semi-annually. If he wants to have a college fund for Huey, Dewey and Louie totaling $300,000 in 10 years, how much should he invest now? (Round your answer to the nearest cent)

15. Money is invested at interest rate r, compounded continuously. Express the time required for the money to quadruple, as a function of r.

16. Two investments are started at the same time. The first investment begins with $5200 and earns interest compounded continuously at the rate of 7.6% per annum. The second account begins with $6400 and earns interest compounded continuously at a rate of 4.3% per annum.
    (a) Determine the time T (in years) for the values of the two accounts to be the same.

    (b) Determine the common value of the accounts at that time. (Round your answer to the nearest cent)

Section 3.6: Modeling with Exponential and Logarithmic Functions

When building an exponential model pay close attention to your initial value (if provided), your rates (is it decreasing or increasing?), units of time (as applicable) and your base (for continuous growth use base *e*).

Write the appropriate equation and then fill in any known values. Use any data points provided to solve for the remaining constants as needed.

Examples

17. If a certain bacteria population triples in 2.7 hours, determine the time T (in hours) that it takes for the population to double.

18. Jennifer is terrified of spiders. Much to her horror, she finds a spider nest in her basement and lays down some poison traps. The population of spiders is supposed to decrease exponentially if the poison works properly. If it takes 12 hours for the first 10% of spiders to die, how long until 75% of the spiders are gone? (Round your answer to the nearest hour)

19. Let $f(x) = 7a^x + c$, where a is positive. If $f(x)$ has a horizontal asymptote at $y - 0.68$ and contains the point (2,4). Determine the values for a and c.

20. Kim still really loves donuts. So much so that she is now making her own tasty creations and uploading videos of the process to her YouTube channel. The number of views of her latest video can be modeled using the following equation:

$$f(t) = 200e^{rt}$$

(a) Should the value of r be positive or negative? Why do you think so?

(b) Kim's donuts are so popular that she decides to have a tasting party. Suppose that instead of video views, $f(t)$ is now modeling the number of donuts in her inventory. Should the value of r be positive or negative? Why do you think so?