By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Points Possible	Points Made
1	0	
2	25	
3	15	
4	10	
5	15	
6	20	
7	15	
Total:	100	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
- Common identities:

$$\begin{aligned}\cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta),\\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: Who do you think will win Dwars door Vlaanderen this year? (one point each for the male and female riders.)

2. Determine all of the values of x for each question below that satisfy the given equation. If no values of x satisfy the equation provide a brief justification as to how you arrived at your conclusion.

_____ (a) [5 pts] $\ln(4x - 1) = 12$.

(b) [5 pts] $\ln(x) - \ln(x - 2) = 3$.

_____

(c) [5 pts] $\left(e^{x/2} - 4\right)^2 = 1.$

(d) [10 pts] $8 \cdot 3^{x+1} = 17 \cdot 4^{1-x}.$

3. The following values for an unknown base, b, are given below:

$$\begin{aligned} \log_b(2) &\approx 0.5664 \\ \log_b(3) &\approx 0.8977 \\ \log_b(5) &\approx 1.3151 \end{aligned}$$

Use this information to determine the exact numerical values of each of the expressions below.

(a) [5 pts] $\log_b\left(\frac{5}{2}\right)$

(b) [5 pts] $\log_b(15)$

(c) [5 pts] $\log_b\left(\sqrt{10}\right)$

4. [10 pts] The questions below refer to the function

$$k(x) \;=\; 2e^{3x-1}.$$

Show that $k(x)$ is a one-to-one function. (Show every algebraic step and do not refer to a graph.)

5. Frohlich and Dobiášová have defined the Atherogenic Index of Plasma[1], called "AIP," in terms of a patient's density of triglycerides (TG) and the density of HDL cholesterol (HDL), as

$$AIP = \log\left(\frac{TG}{HDL}\right).$$

(a) [5 pts] If a patient has a TG level of 110 mg/l and an HDL level of 45 mg/l determine the patient's AIP.

(b) [5 pts] If a patient has an AIP of 0.33 and a TG level of 105 mg/l determine the patient's HDL.

(c) [5 pts] A patient has an HDL level of 47 mg/l, and a healthy AIP for the patient should be below 0.35. Determine the possible range of healthy values of TG for this patient. (Your answer should be in interval notation.)

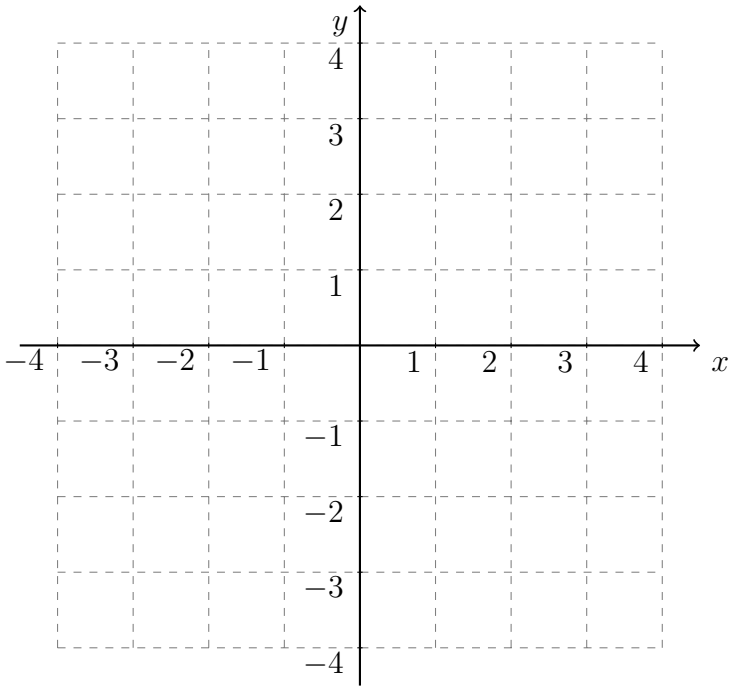
[1]Däg H, Incirkuş F, Dikker O. Atherogenic Index of Plasma (AIP) and Its Association with Fatty Liver in Obese Adolescents. Children (Basel). 2023 Mar 29;10(4):641. doi: 10.3390/children10040641.

6. Two functions will be compared,

$$\begin{aligned} p(x) &= \log_2(x) + 1, \\ m(x) &= \log_2(a \cdot x), \end{aligned}$$

where a is a positive constant ($a > 0$).

(a) [5 pts] Use the axes below to sketch the graph of $p(x)$.



(b) [5 pts] Determine the range and domain of $p(x)$ (Your answer should be in interval notation.)

(c) [5 pts] Determine the range and domain of $m(x)$ (Your answer should be in interval notation.)

(d) [5 pts] For what values of a (if any) is $p(x) < m(x)$ for all x in the domain of both functions? (Your answer should be in interval notation.) Provide a brief justification for your result.

7. A group of Chemistry graduate students were clumsily tossing a bottle of Chemical X, and the bottle broke spilling into Lake Herrick. The lake is closely monitored, and it was determined that after 24 hours there was 0.5 grams of Chemical X in the lake. After a total of 48 hours there was 0.4 grams of Chemical X in the lake. Assuming that the decay of the chemical follows an exponential function answer each of the following questions.

   (a) [10 pts] Determine a function that returns the amount of Chemical X given the number of hours after the initial spill.

   (b) [5 pts] How long will it take for the amount of Chemical X to be reduced to 0.01 grams?

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____