University of Georgia	Math 1113	
Department of Mathematics	Final Exam	Spring 2025

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____   Name (print): _____

Student Number: _____

Instructor's Name: _____   Class Time: _____

Problem Number	Total Points Possible	Points Made
1	0	
2	25	
3	20	
4	20	
5	15	
6	10	
7	15	
8	20	
9	15	
10	8	
11	10	
12	10	
13	12	
14	15	
Total:	195	

- If you need extra space use the last page. *Do not tear off the last page!*

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.

- Please turn off your mobile phone.

- You are only allowed to use a TI-30 calculator. No other calculators are permitted.

- A calculator is not necessary, and answers should be given in a form that can be directly entered into a calculator. If you give a numerical value it should be to within one decimal place unless otherwise stated.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

1. [2 Bonus] Common Knowledge: How will Movistar balance the big egos of Marlen Reusser, Liane Lippert, and Cat Ferguson at the Vuelta España Femenina?

2. Determine all values of x that satisfy each equation below.

   (a) [5 pts] $\dfrac{1}{3x+1} = 4$.

   (b) [5 pts] $\sqrt{x+1} = x$.

(c) [5 pts] $e^{-3x+1} = 4 \cdot e^x$.

(d) [5 pts] $\ln(x+4) + \ln(x-4) = 8$.

(e) [5 pts] $7 \cdot 2^{1-4x} = 3 \cdot 9^{5+x}$.

3. Determine the value of each of the requested quantities below. If an exact number is not requested, numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine its numerical value**.)
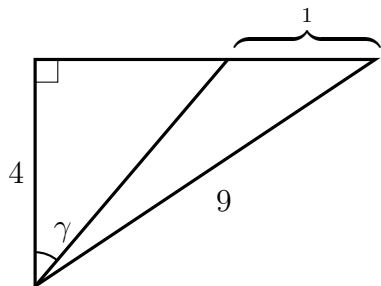
   (a) [5 pts] Determine the sine, cosine, and tangent of the angle α as shown in the diagram below:
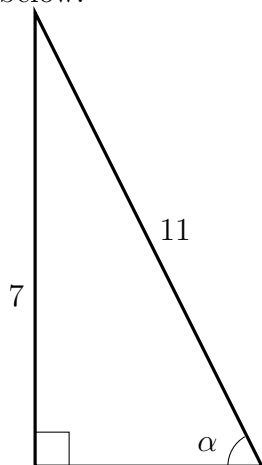
   (b) [5 pts] The radian measure of the angle β in the diagram below is $\dfrac{\pi}{12}$. Determine the value of length b.
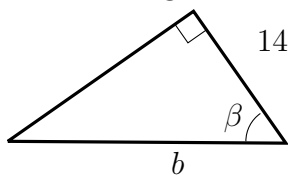
(c) [5 pts] Determine the radian measure of the angle γ in the diagram below.
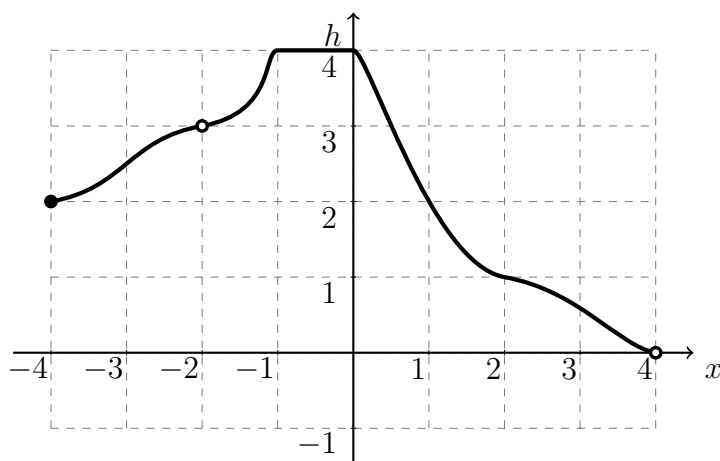

(d) [5 pts] Determine an equivalent expression to

$$\cos\left(\arcsin(x) + \arctan\left(\frac{2}{3}\right)\right).$$

that does not contain any trigonometric functions.

4. The graph of a function, $h(x)$, is shown below, and the function $g(x)$ is defined. Use the two functions to answer each of the questions below.

$$g(x) = \frac{4x}{x+3}.$$

(a) [5 pts] Determine the domain of the function $g(x)$. (Your answer should be in interval notation.)

(b) [5 pts] Determine a restriction on the domain of h so that it is one-to-one on the restricted domain and the range of the function on the restricted domain is the same as the range of the original function, h. (Your answer should be in interval notation.)

(c) [5 pts] Determine the numerical value of $g(h(-1))$.

(d) [5 pts] Determine the numerical value of $h(g(-1))$.

5. Two functions are defined by

$$k(x) = 3\cos\left(2x + \frac{\pi}{2}\right),$$
$$q(x) = 3\sin\left(m \cdot x + \pi\right),$$

where m is a positive, non-zero, constant number.

(a) [5 pts] Determine the amplitude and period of $k(x)$.

(b) [5 pts] Determine the domain and range of $q(x)$. (Your answer should be in interval notation.)

(c) [5 pts] Determine the possible values of m (that are positive) so that the period of $q(x)$ is longer than the period of $k(x)$. (Your answer should be in interval notation.)

6. Each question below refers to the function

$$r(x) \;=\; \frac{1}{x+3}.$$

(a) [5 pts] Show that $r(x)$ is a one-to-one function. (You should provide an algebraic argument and not use a graph or find the inverse.)

(b) [5 pts] Determine the inverse of $r(x)$.

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] The cost of insurance associated with a piece of equipment increases by $50 each month. *The cost of the insurance as a function of the time in months.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

   (b) [5 pts] The monthly cost of insurance associated with a piece of equipment increases by 1% each month. *The cost of the insurance as a function of the time in months.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

   (c) [5 pts] The monthly cost of insurance associated with a piece of equipment increases in the first half of the year, decreases in the second half of the year, and then repeats the cycle. *The cost of the insurance as a function of the time in months.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

8. The price of a part is initially \$2.50. The cost of the part increases by \$0.02 each month for twenty-four months. After the initial twenty-four months the price decreases by \$0.03 each month until the cost is \$2.20, and it remains the same afterwards.

   (a) [10 pts] Determine the cost of the part given the time in months. Define the function using proper piecewise defined notation.

   (b) [5 pts] What is the maximum cost of the part?

   (c) [5 pts] Is the cost a one-to-one function? (Briefly justify your conclusion.)

9. Each of the following questions refer to the function

$$b(x) \ = \ 3x^2 + 6x + 8.$$

(a) [5 pts] Determine the vertex of the parabola. Your answer should be a coordinate, (x, y).

(b) [5 pts] Determine the values of x where the function is increasing.

(c) [5 pts] Determine the average rate of change of $b(x)$ from $x = 0$ to $x = 2$.

10. A logistic function is given by

$$L(x) = \frac{C}{1 + 3e^{-rx}},$$

where C and r are constants.

(a) [5 pts] It is known that $L(0) = 3$ and $L(10) = 5$; determine the values of the constants C and r.

(b) [3 pts] Determine the value the function approaches as the value of x gets extremely large.

11. [10 pts] Determine a formula for the function whose graph is shown below expressed as a cosine function,

$$g(x) \;=\; A \cdot \cos(bx + c) + d.$$

The values of A and b should be positive numbers.
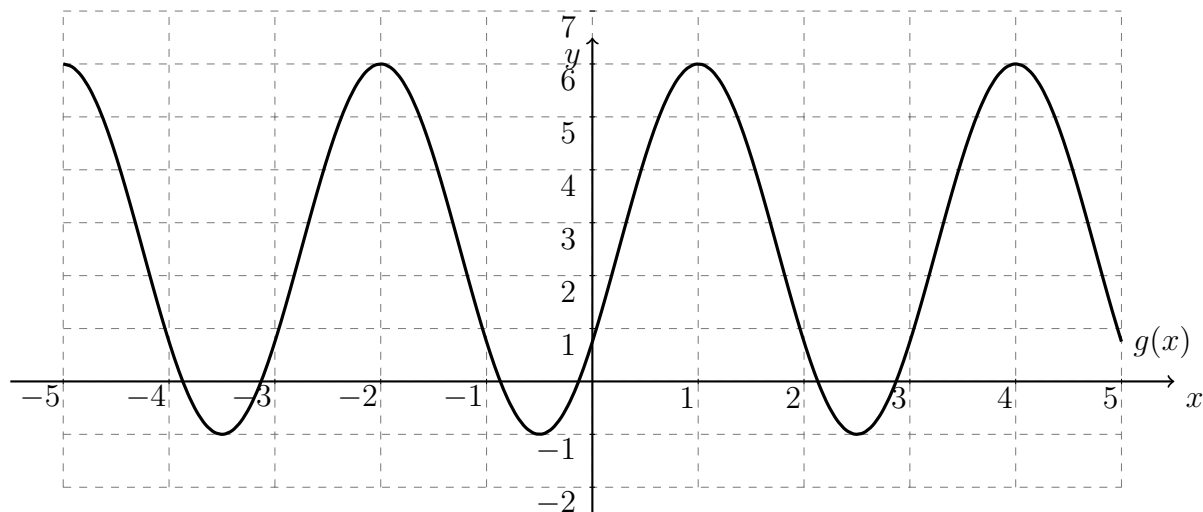
$A =$

$b =$

$c =$

$d =$

12. [10 pts] A guard stands on top of a building. The guard's height is 1.5m, and the height of the building is 84m. The guard is boldly looking straight in the eyes of a world renowned jewel thief, known to be 2m tall, who is standing on the ground away from the building. The guard's angle of depression is 37.5°. How far is the thief standing from the building?

13. It is estimated[1] that the gross domestic product (GDP) tends to depend on the country's population (Pop) by the relationship,

$$\text{GDP} \;=\; C \cdot (\text{Pop})^{\alpha},$$

where C and α are constants. (The variable, Pop, is the population measured in millions. For a population of 100 million Pop is 100.)

(a) [5 pts] A country with a population of one-million has a GDP of \$800,000, and a country with a population of two-million has a GDP of \$1,365,000. Determine the values of C and α.

(b) [5 pts] If a country has a GDP of one billion US dollars, what is the estimated population?

(c) [2 pts] Given your estimate of α does this imply that the GDP per capita (defined to be the GDP divided by the population) is a good way to compare two different countries? (Briefly explain why your answer is consistent with your results above.)

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[1] Jiang Zhang, Tongkui Yu, Allometric scaling of countries, Physica A: Statistical Mechanics and its Applications, Volume 389, Issue 21, 2010, Pages 4887-4896,

14. [15 pts] Farmer Bob has 50 acres and will plant two crops, rutabagas and arugula. The expected probability that there will not be a mold infestation depends on the number of acres planted of each crop.

   Rutabagas The probability there will not be an infestation due to rutabagas is a linear function. The probability is 0.1 if no acres are used for rutabagas, and the probability is 0.2 if 50 acres are used for rutabagas.

   Arugula The probability there will not be an infestation due to arugula is a linear function. The probability is 0.2 if no acres are used for arugula, and the probability is 0.5 if 50 acres are used for arugula.

   Assuming the total probability is the product of the two individual probabilities determine the number of acres to allocate to each crop to maximize the probability there will not be a mold infestation.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____