**University of Georgia**                     **Math 1113**
**Department of Mathematics**           **Test 2**                               Spring 2024

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign):  _____          Name (print):  _____

Student Number:  _____

Instructor's Name:  _____          Class Time:  _____

| Problem Number | Points Possible | Points Made |
|:---:|:---:|:---:|
| 1 | 0 | |
| 2 | 22 | |
| 3 | 15 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 20 | |
| 7 | 10 | |
| 8 | 10 | |
| Total: | 97 | |

- If you need extra space use the last page.

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.

- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.

- Please turn off your mobile phone.

- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

1. [2 Bonus] Common Knowledge: Given the retirement of Annemiek Van Vleuten, which Movistar rider will rise to be the team leader for the 2024 season?

2. Determine all of the values of $x$ for each question below that satisfy the given equation. If no values of $x$ satisfy the equation provide a brief justification as to how you arrived at your conclusion.

_____   (a) [5 pts]   $\ln(3x + 1) = 4$.

(b) [5 pts]   $\dfrac{e^{2x} + 1}{4} = 7$.
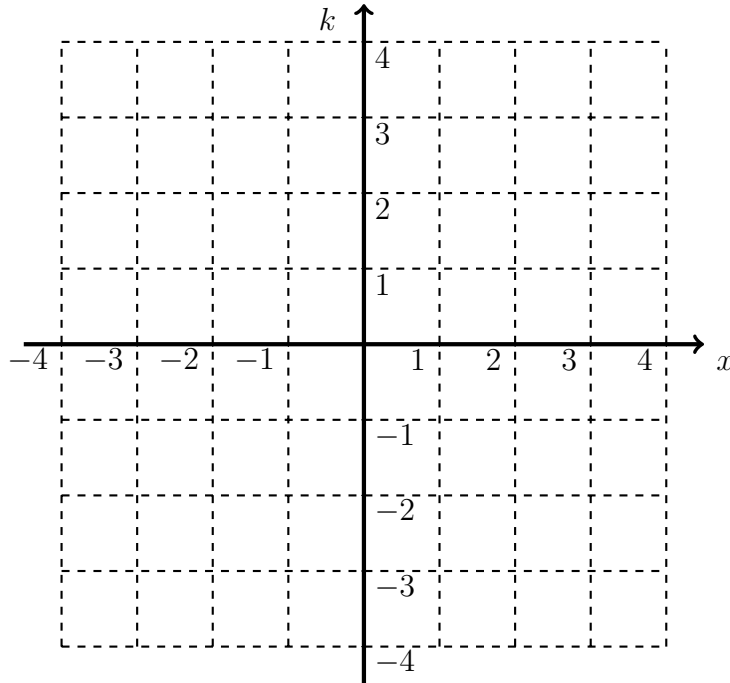
_____

(c) [5 pts]   $\ln(x+4) + \ln(x+1) = 2.$

(d) [7 pts]   $14 \cdot 2^{-x} = 3 \cdot 9^{x+1}.$

Points earned: _____
out of a possible 12 points

3. Given the function

$$h(x) = \log(2x + 1)$$

answer each of the following questions.

(a) [5 pts] Make a rough sketch of the function using the aces below:



(b) [5 pts] Determine the inverse of the function $h(x)$.

(c) [5 pts] Verify your answer to part b by using composition. (In other words, use the definition of the inverse to show that your answer is correct.)

4. Simplify each of the expressions below to an equivalent expression that only has one single logarithmic function.

   (a) [5 pts] $\ln(r+1) - \ln(s) + \ln(3-p)$

   (b) [5 pts] $3\ln(u) + \dfrac{1}{2}\ln(w) - 4\ln(2+q)$

5. Simplify each of the expressions below so to an equivalent expression that only has one single exponential function.

   (a) [5 pts] $\dfrac{e^{4t} \cdot e^{-s}}{e^{4u}}$

   (b) [5 pts] $\left(\dfrac{e^{2p}}{e^{1-s}}\right)^{3} \sqrt{e^{-w}}$

6. [10 pts] A bank offers various options for saving accounts. One account is advertised as having a rate of 1.2% compounded weekly.

   (a) [5 pts] A customer opens an account with the advertised rate with an initial balance of $4,000. How much will be in the account after 24 weeks assuming there are no withdrawals or deposits?

   (b) [5 pts] A different customer is examining the account and asks how many weeks would it take for a balance to increase by %50 (the balance will be 150% of the initial balance). What would an honest, helpful banker tell the customer?

7. The literacy level of a child is measured by counting the number of correct words per minute a child (WCPM) can read aloud. The expected WCPM for a child as a function of their age (in years) is often modelled as a logarithmic function. Suppose that

$$WCPM(t) \;=\; 1500 \log(0.5t + 1.2).$$

(a) [3 pts] What is the expected WCPM for a six year old child?

(b) [5 pts] A child has a WCPM of 1150 correct words per minute. What is the estimated age level the child reading at?

(c) [2 pts] Why is a logarithmic function more appropriate than an exponential function for this model?

8. [10 pts] A patient is given 20 mg of a drug. It is assumed that the amount of the drug in the patients bloodstream decays exponentially, and it is estimated that after three hours there is 15 mg of the drug in the patient's bloodstream. Determine a function that will provide the amount of drug in the patient's bloodstream given the number of hours after given the drug. Use your formula to estimate how long it will take for the amount of the drug to be reduced to 5 mg.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____

Points earned: _____
out of a possible 0 points