

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): \_\_\_\_\_

Name (print): \_\_\_\_\_

Student Number: \_\_\_\_\_

Instructor's Name: \_\_\_\_\_

Class Time: \_\_\_\_\_

Problem Number	Points Possible	Points Made
1	0	
2	20	
3	20	
4	17	
5	20	
6	15	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	11	
14	10	
Total:	173	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
- Common identities:

$$\begin{aligned}\cos(\alpha + \beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta), \\ \sin(\alpha + \beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: What will Marianne Vos have to do to beat Fem Van Empel this cross season?

2. Determine all values of  $x$  that satisfy each equation below.

(a) [5 pts]  $2x^2 + 5x - 7 = 0$

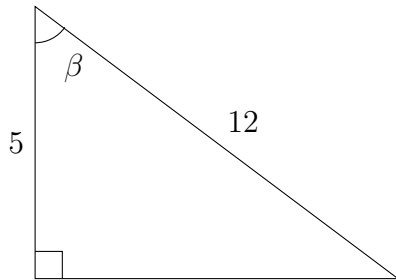
(b) [5 pts]  $\log(4x^2 - 3) = 5$

(c) [5 pts]  $\sqrt{4 - 5x} = 7$

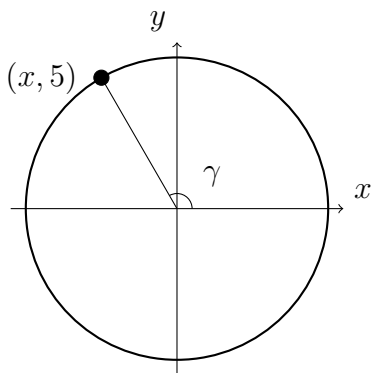
(d) [5 pts]  $2 \cdot 8^{x+1} = 7 \cdot 6^{3-2x}$

3. Determine the value of each of the requested quantities in each question below. Numerical values should be to within 0.01 of the true value. (**All angles are given in radians and should be expressed in radians if you have to determine their value.**)

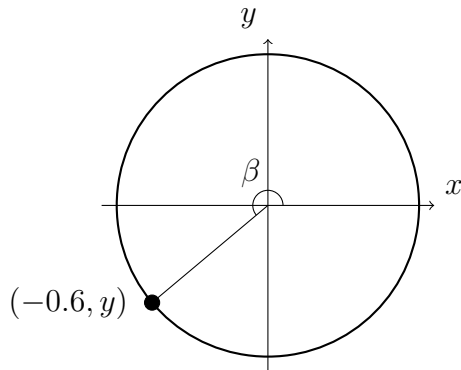
(a) [5 pts] Determine the cosine, sine, and tangent of the angle  $\beta$  in the diagram below.



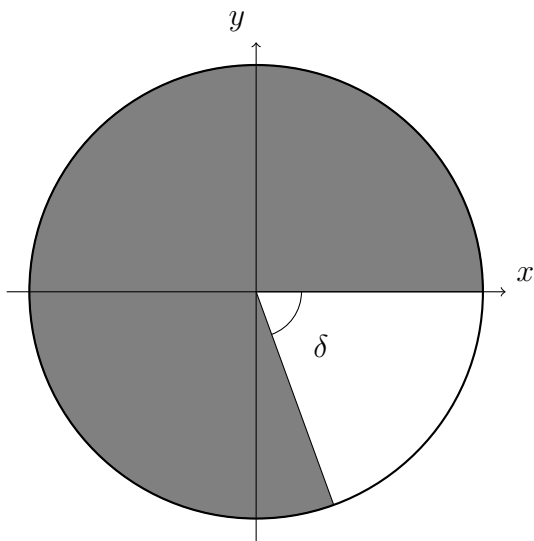
(b) [5 pts] Determine the cosine, sine, and tangent of the angle  $\gamma$  in the diagram below. The radius of the circle in the diagram is 6.



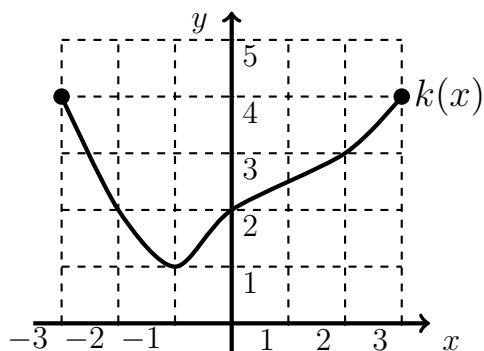
- (c) [5 pts] Determine the radian measure of the angle,  $\beta$ , in the diagram below. The point shown is on the unit circle.



- (d) [5 pts] Determine the radian measure of the angle  $\delta$  in the diagram below. The circle has a **radius of two**, and the **arclength** of the shaded sector is 10.



4. Three functions,  $k(x)$ ,  $w(x)$ , and  $g(x)$ , are given below. Use the functions to answer each of the questions below.



$x$	$w(x)$
0	-2
1	-3
2	0
3	-1
4	2
5	3

$$g(x) = \begin{cases} x & x < 0 \\ -x + 3 & x > 0 \end{cases}$$

- (a) [5 pts] Determine the value of  $w(k(-1))$

- (b) [5 pts] Determine the value of  $w(g(5))$

- (c) [4 pts] Determine the average rate of change of  $g(x)$  from  $x = -2$  to  $x = 4$ .

- (d) [3 pts] For what values of  $x$  is the function  $g(k(x))$  increasing?

5. For each description below determine a formula for an equation that matches the requirements.

(a) [10 pts] A parabola that is increasing if  $x < 4$  and decreasing if  $x > 4$ . The parabola has a maximum value of  $-1$ . The  $y$ -intercept of the parabola is at  $y = -10$ .

(b) [10 pts] A linear function that goes through the point  $(2, 5)$ , and the line is parallel to the line  $3x - 8y = 7$ .

6. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

- (a) [5 pts] Money is deposited into a bank account that offers compounded interest, and no money is withdrawn. *The amount of money in the bank account as a function of time*

Linear  
Function

Quadratic  
Function

Exponential  
Function

Trigonometric  
Function

- (b) [5 pts] A bird lands on the end of the blade of a windmill and remains in place. *The bird's height as a function of time*

Linear  
Function

Quadratic  
Function

Exponential  
Function

Trigonometric  
Function

- (c) [5 pts] A Coast Guard ship is sent to look for a person who was swept off of a boat. The captain of the ship specifies a search area that is a square whose sides grow at a constant rate in time. *The area in square meters of the search area as a function of time*

Linear  
Function

Quadratic  
Function

Exponential  
Function

Trigonometric  
Function



7. [10 pts] An object is raised along a wall. The height of the object increases at a constant rate. Initially the object is 10 meters above the ground, and after 120 seconds the object is 30 meters above the ground. Determine the height of the object above the ground as a function of time for any time between 0 and 120 seconds.

8. [10 pts] A field has a rectangular shape, and one of its dimensions is 40 meters. A cord will be stretched across the diagonal between two opposite corners of the field. Determine the length of the cord given the area,  $A$ , of the field. (Your answer should be a function of  $A$ .)

9. [10 pts] Determine the value of

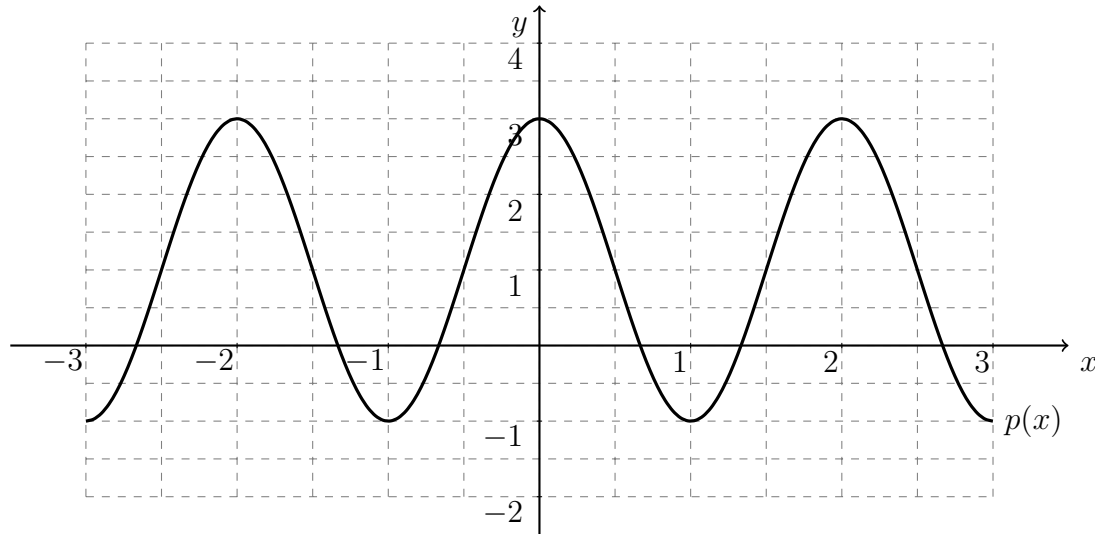
$$\cos(\arctan(2.3) + \arccos(0.2)).$$

Express the **exact** value so that no trigonometric functions occur in the expression. (Do not use a calculator, and you do not need to simplify your expressions.)

10. [10 pts] Determine a formula for the function whose graph is shown below expressed as a sine function,

$$p(x) = A \sin(bx + c) + d.$$

The values of  $A$ ,  $b$ , and  $c$  should be positive numbers.



$$A =$$

$$b =$$

$$c =$$

$$d =$$

11. The amount of mercury, in parts per million (ppm), that will accumulate in a fish at a given time,  $t$  years, after it is put in a stream is expected to be given by the function

$$M(t) = A - 0.15e^{rt},$$

where  $A$  and  $r$  are constants. It is estimated that a fish initially has 0.05 ppm of mercury, and after one year is expected to have 0.09 ppm.

- (a) [5 pts] Determine the function that gives the amount of mercury in the fish at any time. (Determine the values of the constants.)

- (b) [5 pts] How long will it take for the amount of mercury to accumulate to a level of 0.17 ppm?

12. [10 pts] Verify the following identity,

$$\cos(x + h) - \cos(x - h) = -2 \sin(x) \sin(h).$$

13. The number of protozoa in a sample is expected to follow logistic growth. The density of organisms (number per cubic millimeter) is given by the equation

$$D(t) = \frac{A}{1 + 10e^{-rt}},$$

where  $A$  and  $r$  are constants, and the time,  $t$ , is the number of days since the sample was made. It is estimated that there are initially 2,000 protozoa per cubic millimeter, and after four days there are 2,300 protozoa per cubic millimeter.

- (a) [8 pts] Determine the formula for the number of protozoa at any given time. (Determine the values of the constants.)

- (b) [3 pts] What will the density get close to after a very long time? (If you are not confident about your answer to the first part above, use the incorrect values of  $A = 500$  and  $r = 3$  for this part.)

14. [10 pts] Two concrete patios will be constructed, and each one will be in the shape of a square of differing dimensions. Both of the patios will have a fancy fence around them, and a total of fifty meters of fencing will be used. The cost of the first patio is ten dollars per square meter of concrete, and the cost of the second patio is fifteen dollars per square meter of concrete. Determine the dimensions that will minimize the total cost of the concrete used.



Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): \_\_\_\_\_ Instructor (print): \_\_\_\_\_ Time: \_\_\_\_\_