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MATH 1101 Exam 2 Review

Spring 2018

Topics Covered

Section 3.4 Linear Regression: Finding the Line of Best Fit

Section 5.1 Exponential Growth Functions

Section 5.2 Exponential Decay Functions

What's in this review?

1. Review Packet

The packet is filled in along with the instructor during the live review. If you missed the live review, see the video link at either web address at the top of the title page for the video link to watch the review.

2. Calculator Screen Shots/Useful Formulas

For your convenience, selected screen shots are available as a quick reference. The video shows the calculator screen for specific problems in the packet.

3. Practice Problems

The problems are meant for you to try at home after you have watched the review. The answers are provided on the last page. There are tutors in Milledge Hall and Study Hall to help you if needed.

Section 3.4 Linear Regression: Finding the Line of Best Fit

Continued from Last Time...

How do we work with functions or data that are 'almost' linear?

If the data points show that the function behaves linearly but doesn't have a constant ARC, we can use Linear Regression to find the line of best fit that will approximate a model for us.

Linear Regression

A computational technique that produces a linear model that best fits the data set. The line of best fit is determined by minimizing the SSE (sum of squared errors) and AE (average error)

Model Error

Since the actual data points may or may not lie on the line of best fit, the model error for a specific data point is calculated by finding the vertical distance between the point and the line.

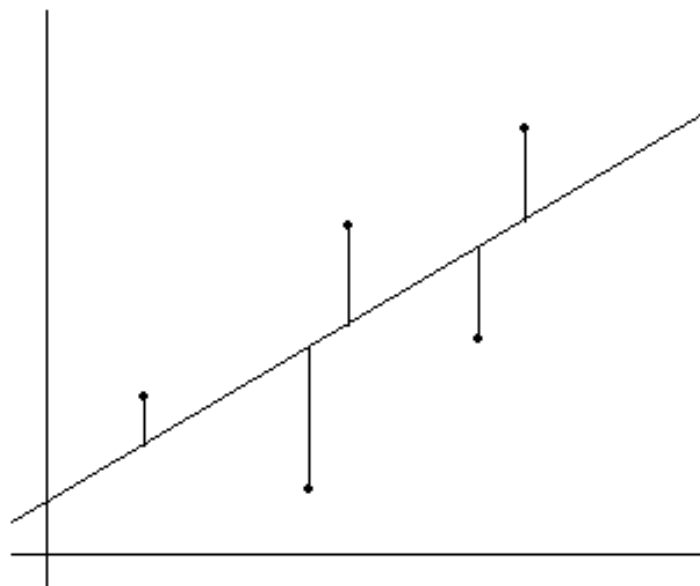
$$\text{Error} = \text{actual value} - \text{predicted value}$$

The actual value is the y value of the data point for a given x and the predicted value is the y value the model gives for the same value x .

- Data points that lie above the line will produce positive errors and the model under predicts
- Data points that lie below the line will produce negative errors and the model over predicts
- The average error (AE) of the model is found by using the SSE (Sum of Squared Errors) where n is the total number of data points in the set

$$AE = \sqrt{\frac{SSE}{n}}$$

What does it look like?



Finding the Line of Best Fit in the Calculator

1. Go to STAT→Edit...
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Go to STAT→CALC→LinReg(ax+b)
5. Type L_1, L_2, Y_1 (Y_1 is under VARS)
6. You will see the values for a, b and the line will be stored in Y_1

Example 1

Find the line of best fit for data set 1.

Data Set 1							
x	63	64	66	69	71	73	75
y	127	121	142	162	156	181	208

Finding the SSE in the Calculator

1. Go to STAT→Edit...
2. Enter the x values in L_1
3. Enter the y values in L_2
4. Enter $Y_1(L_1)$ in L_3
5. Enter $L_2 - L_3$ in L_4
6. Enter L_4^2 in L_5
7. Go to 2^{nd} →STAT→MATH→sum(L_5)

Example 2

Find the SSE and AE for data set 1. Round your answers to 2 decimal places.

SSE:

AE:

Percent Error

We can use the AE to calculate the % error for a specific data point. $\frac{AE}{y} \times 100\% = \% \text{ error}$

Example 3

Find the percent error for $x = 63$ and $x = 75$ for data set 1.

Interpolation vs. Extrapolation

The line of best fit can ONLY predict y values for x values that are between the x_{min} and x_{max} in the original data set. This is called interpolation. Extrapolation are attempts to predict y values that lie outside the interval $[x_{min}, x_{max}]$. The model can NEVER extrapolate!

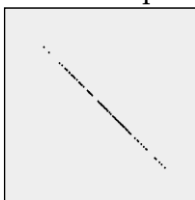
Example 4

Which of the following values can yield predictions from the model for data set 1? Give the predicted value and error for the ones you can. {60, 65, 68, 72, 80}

Correlation Coefficient, r

The correlation coefficient measures the *strength* of the relationship of the two variables used in the model. r can be any value in the interval $[-1,1]$. The sign of r matches the sign of the slope.

Below is a visual representation of r



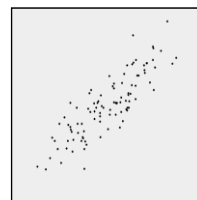
$r = -1$



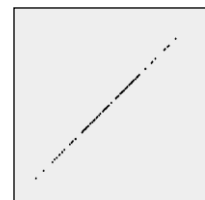
$r = -.50$



$r = 0$



$r = +.85$



$r = +1$

You can see the r value for any model you build using LinReg by turning the DiagnosticsOn.

Example 5

What is the correlation coefficient for the model for data set? What can we say about the two variables in the model?

Example 6

In data set 1, if x is the number of people in a room and y is the total of one dollar bills in their pockets, how many people should be in the room to have \$150 in one dollar bills?

Section 5.1 Exponential Growth Functions

Exponential Functions

A function is called exponential if the variable appears in the exponent with a constant in the base.

Exponential Growth Function

Typically take the form $f(x) = ab^x$ where a and b are constants.

- b is the base, a positive number > 1 and often called the *growth factor*
- a is the y intercept and often called the *initial value*
- x is the variable and is in the exponent of b

Growth Factor, b

b is the growth factor and constant number greater than 1. Therefore it can be written in the form:

$$b = 1 + r$$

Where r is the growth rate. The rate is a percentage in decimal form. An alternative form of the exponential growth function is:

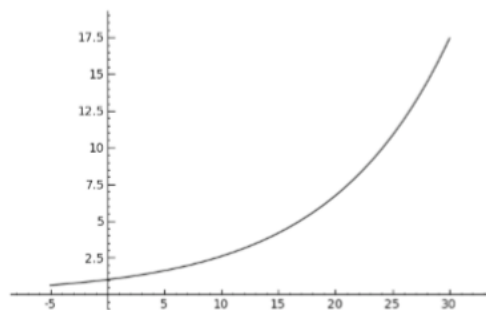
$$f(x) = a(1 + r)^x$$

Linear vs. Exponential Growth

If a quantity grows by 4 units every 2 years, this describes a linear relationship where the slope is $4 \text{ units}/2 \text{ years} = 2 \text{ units/year}$

If a quantity grows by a factor of 4 units each year, this describes an exponential relationship and the quantity increases 300% each year. $4 = 1 + 3.00$

Graphs of exponential growth functions are concave up.



Exponential growth functions ($b > 1$)
increase at an increasing rate.

Example 7

On January 1, 2000, \$1000 was deposited into an investment account that earns 5% interest compounded annually. Answer the following:

- (a) Find a model $A(t)$ that gives the amount in the account as a function of t . Let t be the number of years since 2000.

- (b) Assuming no withdrawals, how much money will be in the account in 2010?

- (c) How long until the account has a balance of \$1575?

- (d) What year and month did the balance hit \$1575?

- (e) How long does it take for the account to double in value?

Example 8

How much money must be invested in an account that earns 12% a year to have a balance of \$3000 after 4 years? Round your answer to the nearest cent.

How long with it take for the account to double in value?

Section 5.2 Exponential Decay Functions

Exponential Functions

A function is called exponential if the variable appears in the exponent with a constant in the base.

Exponential Decay Function

Typically take the form $f(x) = ab^x$ where a and b are constants.

- b is the base where $0 < b < 1$ and often called the *decay factor*
- a is the y intercept and often called the *initial value*
- x is the variable and is in the exponent of b

Decay Factor, b

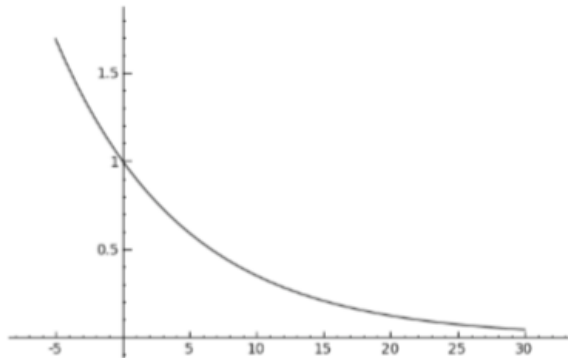
b is the decay factor and constant number between 0 and 1. Therefore it can be written in the form:

$$b = 1 - r$$

Where r is the decay rate. The rate is a percentage in decimal form. An alternative form of the exponential decay function is:

$$f(x) = a(1 - r)^x$$

Graphs of exponential decay functions are concave up.



Exponential decay functions ($0 < b < 1$)
decrease at a decreasing rate.

Example 12

Determine the model $f(t)$ for a substance that decays at 12% per hour and an initial value of 2500.

Example 13

The town of Pawnee had a population of 242.3 thousand in 2005 and began decreasing at a rate of 0.3% after Little Sebastian was kidnapped by Eagleton residents during Pawneefest that year. Assuming this trend continues for several years, answer the following:

- (a) Find a model $P(t)$ that gives the population of Pawnee as a function of t , the number of years since 2005.

- (b) Predict when the population of Pawnee will fall to 236 thousand.

- (c) Predict the population of Pawnee in 2011.

- (d) What is the annual decay factor?

- (e) The 10 year decay factor?

- (f) The 10 year growth rate? How do we interpret this value?

- (g) What is the monthly growth rate?

Example 14

The town of Eagleton had a population of 185 thousand in 2005 and a population of 162 thousand in 2010 after the city coffers went bankrupt. Answer the following.

- (a) Find a model $P(t)$ that gives the population of Eagleton as a function of t , the number of years after 2005.

- (b) What is the decay rate of Eagleton's population?

- (c) When will Eagleton's population fall to 100 thousand?

Useful Calculator Screenshots

Calculator Tip #14

A set of data points $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots$ can be stored in the calculator.

- Store x_1, x_2, x_3, \dots in list **L1** (see tip #13)
- Store y_1, y_2, y_3, \dots in list **L2** (see tip #13)

Try it: Store the following data points in the calculator.

Year t	1980	1990	2000	2010
Population P (thousands)	366	437	529	584

24



In **STAT** Edit
with **L1** and **L2**
ready for input

25

Calculator Tip #17

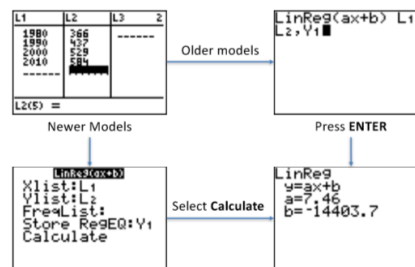
The best-fitting linear model $f(x) = ax + b$ for a set of data points $(x_1, y_1), (x_2, y_2), (x_3, y_3), \dots$ can be found.

- Store x_1, x_2, x_3, \dots in list **L1** (see tip #13)
- Store y_1, y_2, y_3, \dots in list **L2** (see tip #13)
- Press **STAT** **CALC** **LinReg(ax+b)**
- For older models:
 - Type **L1,L2,Y1**
 - Press **Enter**
- For newer models:
 - Set **Xlist = L1**, **Ylist = L2**, and **Store RegEQ = Y1**
 - Select **Calculate**

Try it: Find the linear regression model $P(t) = at + b$ for these data.

Year t	1980	1990	2000	2010
Population P (thousands)	366	437	529	584

29



Y1 is located at **VARS** **Y-VARS** **Function**;
L1 and **L2** are above the **1** and **2** keys

30

Useful Formulas

$$\text{error} = \text{actual} - \text{predicted}$$

$$AE = \sqrt{\frac{SSE}{n}}$$

$$f(t) = ab^x$$

$$f(t) = a(1+r)^t, \quad f(t) = a(1-r)^t$$

$$A(t) = A_0 \left(1 + \frac{r}{n}\right)^{nt}$$

$$EAY = r = \left(1 + \frac{APR}{n}\right)^n - 1$$

$$A(t) = A_0 \left(\frac{A_N}{A_0}\right)^{\frac{t}{N}}$$

Additional Practice Problems

1. Is this a linear function, exponential function, some other type of function, or not a function?

x	0	1	2	3	4	5
y	3.500	.350	.954	1.956	2.80	3.54

2. Is this a linear function, exponential function, some other type of function, or not a function?

x	0	1	2	3	4	5
y	7.100	3.950	.800	2.350	5.500	8.650

3. Is this a linear function, exponential function, some other type of function, or not a function?

x	0	1	2	3	4	5
y	3.100	3.720	4.464	5.357	6.428	7.7137

4. Is this a linear function, exponential function, some other type of function, or not a function?

$$y = 934(1 + 0.39)^x$$

5. Is this a linear function, exponential function, some other type of function, or not a function?

$$y = 27(1 + 3x)^2$$

6. Makebelievia Inc. is making automatic homework doers, they can make 2000 the first year, increasing by 5% per year.

- (a) What is the growth rate?
- (b) What is the growth factor?
- (c) Find a model, $D(t)$ for the production level, as a function of the number of years after the first production.
- (d) Find the amount produced 7 years later.
- (e) Find the time when they first produce over 3000. (Watch your rounding)
- (f) Find the decadal growth rate (growth rate over a decade).

7. You invest \$1700 in a Ponzi scheme paying 3.7% compounded annually.

- (a) Find a model $A(t)$ that gives the amount you believe you have, in dollars, as a function of years after initial deposit.
- (b) Find the amount in the account after 27 years.
- (c) Find the doubling time.
- (d) Find the monthly growth rate.

8. A colony of ants has moved into your roommate's sock drawer, on Feb. 17 the population was 1200, by Feb 20 the population had grown to 2300. Assume exponential growth.

- (a) Find a model $P(t)$ that gives the number of ants as a function of days after Feb. 17th.
- (b) Find the amount of ants on Feb 29.
- (c) Find the day when the number of ants is exactly 3000. (Extra credit: find the hour in the day)
- (d) Find the daily growth factor.
- (e) Find the daily growth rate.

9. Makebelievia's national reserves, in billions, can be modeled by the function $D(t) = 273(.8432)^t$ where t is the number of years after January 1, 2000.

- (a) Find the country's annual percent growth/decay rate.
- (b) The country's reserves [increase / decrease] by _____ each _____
- (c) Find the per-decade growth/decay factor, round to four decimal places (extra credit, why four places?)

- (d) Find the average rate of change during President Phakie's term, (2002 to 2009)
 - (e) The country's actual reserves in 2009 were \$54.71 billion, find the model's error.
 - (f) Predict the reserves in 2018.
 - (g) In what year will the reserves reach 4,380 million? (watch your units)
10. My laptop battery loses 4% of its runtime every month, find the time when it's down to half of its initial runtime.
11. The following data represents the number of points scored in the Makebelievia national championship mudslinging match for several years.

Year	2003	2004	2005	2006	2007	2008
Points	103	131	143	172	184	219

- (a) Find the linear model $M(t) = at + b$ that best fits these data where t is the number of years after 2003.
 - (b) Find the SSE of your model
 - (c) Find the AE (average error)
 - (d) Find the correlation coefficient
 - (e) Can you be 95% confident that there's correlation between year and points? (Pretend you know $r_5 = 0.878$)
 - (f) The model [under-predicts / over-predicts / exactly predicts] the points in 2006
 - (g) Predict the number of points in 2011
12. A group of bacteria doubles in size in 8 hours, if there are 1600 at noon, how many are there at 5:00 pm.
13. Find the effective annual yield for each of these accounts:
- (a) \$1000 invested at 5% compounded annually
 - (b) \$2172 invested at 4.5% compounded daily
 - (c) \$829 invested at 3.2% compounded monthly

Answers:

1. Some other type of function
2. Linear function
3. Exponential function
4. Exponential function
5. Some other type of function
6. (a) 5%
(b) 1.05
(c) $D(t) = 2000(1 + .05)^t$
(d) 2814
(e) $t = 8.31$, but round up to $t = 9$
(f) 62.88%
7. (a) $A(t) = 1700(1 + .037)^t$
(b) \$4533.96
(c) $t = 19.08$
(d) 0.30%
8. (a) $P(t) = 1200(2300/1200)^{t/3}$ or $P(t) = 1200(1.2422)^t$
(b) 16195
(c) $t = 4.23$ or Feb 21, 5:24 am
(d) 1.2422
(e) 24.22%
9. (a) -15.68%
(b) The country's reserves **decrease** by 15.68% each year.
(c) factor: .1631, rate: -83.86%
(d) -19.33 billion/year
(e) -4.12 billion
(f) 12.67 billion
(g) $t = 24.23$ or 2024
10. $t = 16.98$ or about 17 months
11. (a) $M(t) = 21.94t + 103.81$
(b) 143.28
(c) 4.89
(d) 0.9916
(e) Yes
(f) The model **under-predicts** the points in 2006
(g) 279.35
12. 2468 (round to a whole number)
(b) (a) 5%, (b) 4.60%, (c) 3.25%