Carl F. Kossack Calculus Prize Examination
April 3, 2017

Name: ____________________

Instructions

- This test has eight problems, and you have two hours to complete it.
- Fill out your answers in the blank space provided. You may use the back sides of pages.
- **No aid of any kind** is allowed. Calculators are not allowed.
- Show your work, and give clear reasoning.

Good luck! The questions start on the next page.

Grading

Problem #	Points	Score Earned
1	10	
2	10	
3	10	
4	15	
5	10	
6	15	
7	15	
8	15	
Total	100	

Problem 1 (10 points)

An airplane at an altitude of 6400 feet is flying horizontally away from an observer on the ground. At the instant when the angle of elevation is 45° the angle is decreasing at a rate of 0.05 radians/sec. What is the speed of the airplane at that instant?

Problem 2 (10 points)

Find the point on the curve

$$xy - \frac{y^2}{2} = x^3$$

which is not $(0, 0)$ where the graph has a vertical tangent.

Problem 3 (10 points)

Find

$$\lim_{x\to\infty}\left(\frac{1}{\sqrt{x^2+2x}-\sqrt{x^2-x}}\right)$$

Problem 4 (15 points)

Is the quantity $\frac{e}{3} \cdot \ln(3)$ greater than, less than, or equal to 1? Prove your answer.

Problem 5 (10 points)

Let

$$f(x) = \begin{cases} |x|^{3/2} \sin\left(1/x\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Use the definition of the derivative to compute $f'(0)$, and explain your steps.

Problem 6 (15 points)

Suppose $f(x)$ has a continuous second derivative, and it intersects the line $y = mx + b$ at three points. Prove that $f''(c) = 0$ for some c.

HINT: Use the Mean Value Theorem multiple times.

Problem 7 (15 points)

A three dimensional solid is produced by rotating the region bounded by $y = xe^x$, the x axis, and $x = 2$ about the line $x = -1$. Determine the resulting volume.

Problem 8 (15 points)

What are the dimensions of a right circular cone with maximum volume that can be inscribed in a sphere of radius R?